LINEAR LOCATION ESTIMATORS: THE DEPENDENCE OF THEIR QUALITY ON THE SHAPE OF THE PROBABILITY DENSITY FUNCTION, AND THEIR ROBUSTNESS.

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p16 , line 2^-	"at ± K." should be "at ± K (assuming μ=0)."
p17 , line 6	"+ $\int_K^\infty$ " should be "+ $2\int_K^\infty$ "
p45 , line 3	"(5,8),(8,8) and (8,9)" should be "(4,8),(7,8) and (7,9)"
p51 , Table 3.2.2, entry at (1.5,0.16)	"5.496 & 95.071" should be "95.496 & 95.071"
p59 , Table 3.2.10, entry at (1.625,0.055)	".499 & 97.584" should be "97.449 & 97.584"
p67 , line 2^-	"good-of-fit" should be "goodness-of-fit"
p94 , line 4	"variance c" should be "covariance c"
p97 , line 9	" $\frac{1}{n}$ " should be " $\frac{1}{n^2}$ "
p105, lines 7 and 8	"Except for the first two distributions" should be "Except for the first two distributions and the last distribution"
p111, line 2	"receiving positive weight" should be "receiving significant positive weight"
p129, line 2^-	"fo" should be "to"

CONTENTS

APPENDIX PAGE

ABSTRACT

The dependence of the performance of linear location estimators on the shape of the probability density function is investigated. Density functions are classified by their shape, and it is seen how the form and the variance of the best linear location estimator for a distribution varies as the shape of the density function varies. Further, in an attempt to understand why a particular estimator is best for a given shape of density function, the estimation of certain simple distributions ("step" distributions) is considered.

The classification of distributions by their shape allows the robustness of various linear estimators to be tested over an organized and representative set of distributions. Consideration is given to the merits of various measures of the robustness of an estimator and a new measure is proposed. It is found that the assessment of the robustness of an estimator requires consideration of factors which have hitherto been neglected.

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CHAPTER 1

INTRODUCTION

A robust estimator should perform well for a variety of error distributions. However it seems to me that the robustness of estimators may not have been tested over a comprehensive enough set of distributions. In this thesis probability density functions are classified according to their shape, and the robustness of location estimators is tested over a set of distributions in which the shape of the density function is systematically varied. Also a general understanding is sought of the effect of the shape of the density function on estimation - what sort of estimator is best for different shapes of density function and why? Also, in considering robustness, attention is focused on how the quality of an estimator is measured and it is suggested that some of the measures used in the past neglect certain relevant factors.

It is hoped that the thesis fills some of the gaps referred to by Bickel in the report on the Princeton robustness study (Andrews et al. 1972, p259) when he commented: "It seems to me that one thing these results call for is a more extensive inquiry into what it means for one shape (scale family) to be 'longer tailed' (harder to estimate location for) than another. It might then be easier to see how representative our selection really was". Huber also comments in this report (p254) that "the selection of distributions may not be comprehensive enough".

The main thing about this study that is different from comparable studies is the set of distributions considered. In order to be able to vary the shape of the density more freely the constraint of using probability density functions describable by simple mathematical formulae was dropped and piecewise-linear density functions of the shapes desired were constructed. The idea was to classify the densities according to their shape and to map the best linear estimators over the whole set of densities as the shape was varied. The usual approach is to consider the performance of estimators on various standard distributions. However this can lead to a series of isolated results which cannot easily be combined to give an overview. It was hoped that classifying the shape of density functions would produce a notion of the nearness of densities which was such that if two densities are near one another estimators perform similarly on them. Thus a good estimator could be found for any distribution by fitting it into the classification scheme and using an estimator that was robust in that region. Also it seems that in a practical situation an assumption about the general shape of the error density function could be made more readily than an assumption about the precise mathematical form of the density function. Note that all distributions considered are symmetric.

The other thing that is different about this study is the measure of the quality of an estimator, that is used in considering robustness. Analyzing what factors affect the way in which various measures rate the quality of an estimator, sheds light on just why one distribution is harder to estimate location for than another. (In one sense, being longer-tailed actually means a distribution is *easier* to estimate location for.) It was found that some measures do not allow for certain relevant factors. Other measures which are claimed to be better, are used. One of the measures used, allows study of the estimability of distributions.

The study is restricted to point estimation of a location parameter and the estimators considered have been restricted to linear combinations of order statistics.

In Chapter 2 the system for classifying probability density functions (especially near-normal ones) by their shape is described. The best linear combination of order statistics for estimating the location parameter is calculated for distributions having various values in the classification scheme, and the patterns of variation in these estimators and in their variances are discussed. The merits of various ways of measuring the quality of an estimator in assessing its robustness, are also discussed.

In Chapter 3 I consider the robustness of linear combinations of order statistics over a set of distributions whose densities have a variety of shapes. Different measures of quality are used and further attention is given to the value of the measures themselves.

In Chapter 4 an attempt is made to build an intuition about location estimation. The best linear combinations of order statistics for certain simple distributions ("step distributions") are calculated.

The features of the distributions are varied in an attempt to find how the shape of a density influences the form of the best location estimator. Consideration is then given to understanding the estimation of other distributions. The estimability of distributions is also considered.

The work is exploratory in nature and further testing and development is needed.

CHAPTER 2

THE DEPENDENCE OF THE BEST LINEAR ESTIMATOR ON THE SHAPE OF THE DENSITY FUNCTION FOR NEAR-NORMAL DISTRIBUTIONS

2.1 Introduction

Consider the problem confronting a scientist of finding a single value for a quantity to summarize the information in some observations of the quantity where each of the observations is subject to error. The quality of a method of doing this (i.e. of an estimator) depends on the sort of errors to which the observations are subject.

Thus we must explore the relationship between the quality of an estimator and the probability distribution describing the errors in the data. The ideal would be to know which estimator is best for each error distribution and, for a particular estimation problem, to know the error distribution exactly.

However, in practice, the error distribution is not known exactly. In the classical approach to estimation the difficulties caused by this were not faced. The best estimator was found for some mathematically tractable distribution and it was assumed that this estimator was also good for distributions which were similar to this distribution. This approach is now considered to be unsatisfactory because, while the mean is the best estimator for the normal distribution, it is quite a poor estimator for some distributions only slightly different from the normal distribution.

In robust statistics it is accepted that the error distribution is not known exactly and estimators are sought whose performance is good not only for a single error distribution but also for a neighbourhood of nearby distributions. Such an estimator is described as robust.

Research in robust statistics has concentrated on distributions which are like the normal distribution but which have slightly higher tails. For such distributions estimators which give little weight to the extreme observations are now believed to be robust.

In robust statistics much emphasis has been put on the tail length of the distribution as the feature which largely determines the estimator to use on it. However it now seems that this is not completely satisfactory. In this chapter I attempt to organize the set of distributions more systematically and to find a better set of features of a distribution for determining a good estimator to use on it. Best linear combinations of order statistics and their variances are found. Measures of the quality of an estimator are also discussed.

2.2 The Measure of Quality of an Estimator

The most common measure of the quality of an estimator on a single distribution is its sampling variance. Thus it seems that one natural measure of the robustness of an estimator is the sampling variance it can guarantee over the whole set of distributions likely to have generated given data. In this section it is suggested that some common measures of robustness which involve sampling variance differ from this measure because of two factors which they do not allow for. It seems that the precise choice of measure can make a considerable difference to robustness results: the definition of robustness is not robust.

Over the set of distributions with constant variance, σ^2, and finite first moment, the mean is the minimax estimator with respect to sampling variance. (This is easily seen. The sampling variance of the mean is σ^2/n for any of the distributions. Also the mean is the essentially unique uniformly minimum-variance unbiased estimator for the normal distribution. Thus the mean can guarantee a variance of σ^2/n and no other estimator can achieve a variance of σ^2/n for the normal distribution. Thus the mean is the minimax estimator.)

How can this be reconciled with the fact that the mean is not considered a robust estimator? Consider for example a result of Huber (1964). Using sampling variance as his measure of quality, he found the minimax M-estimator for the set of distributions whose cumulatives are everywhere within ε of the standard normal cumulative. It was not the mean. The fact that the minimax estimator obtained by Huber is different from the minimax estimator obtained above is explained by the fact that the distributions considered by Huber do not all have the same variance whereas those above do. Clearly for Huber's set of distributions the estimator which minimaxes the sampling variance needs higher effectiveness on the distributions with higher variance (where effectiveness is defined as (variance of distribution/variance of estimator), i.e. the inverse of the variance of the estimator when the distribution is scaled to have variance one.)

Thus, to guarantee a small sampling variance for an estimator over a set of distributions the variation in the variances of the different possible distributions must be allowed for. If confronted with data the statistician should not say: this data could have come from distribution family A or distribution family B, so use estimator E which has effectiveness K on each of these. Rather he should say: this data could have come from a distribution from family A with variance 1 or a distribution from family B with variance 1.5, so use estimator E' which is more effective on family B than on family A.

There is a second factor which must be considered if a small sampling variance over a whole set of distributions is required.

In some approaches to robust statistics the quality of an estimator on a distribution is measured relative to the best estimator available for that distribution. The efficiency of an estimator is defined as (variance of best available estimator/variance of estimator). The Princeton study (Andrews et. al. 1972), for example, used a measure based on efficiency (namely deficiency, which equals (1-efficiency)). To guarantee the best efficiency over a set of distributions is not necessarily to guarantee the best sampling variance. This measure fails to allow for the different estimabilities of the different distributions (where by the estimability of a distribution is meant (variance of distribution/variance of best estimator)). If either of two distributions with the same variance could be generating given data and if one is more estimable than the other then we don't require an estimator to be as efficient on the first as on the second in order for it to achieve the same sampling variance.

Measures based on efficiency also fail to allow for the first of the above factors: the variation in the variances of the possible distributions. If either of two distributions which are equally estimable could be generating given data and if one has smaller variance than the other we don't require an estimator to be as efficient on the first as on the second in order for it to achieve the same sampling variance.

For high tailed distributions the two factors above work in opposite directions and may partly cancel one another. Assume we have data which is normal-like in the middle and which may be normally distributed or may have outliers and come from a higher-tailed distribution. The higher the tail the bigger the variance of the distribution and so the more effective we require an estimator to be to achieve the same sampling variance. On the other hand the higher the tail the more non-normal the distribution is and so, as we shall see later, the more estimable it is. Thus the higher-tailed distribution partly compensates for having a bigger variance by being more estimable so that an estimator with the same efficiency for several distributions may have approximately the same sampling variance for all of them. Nevertheless it is desirable that attention be given to these factors and that they be consciously considered.

In this study the quality of an estimator will be measured by (variance of estimator/variance of distribution) or, equivalently, the variance of the estimator on the distribution scaled to have variance one. This measure is a basic one from which other measures could be derived: it includes the raw information using which specific allowance could be made for different distributions having different variances and different estimabilities. Furthermore it is of interest in itself because it measures the estimability of a distribution: if two distributions have the same variance when is one able to be estimated more accurately than the other?

2.3 The Organization of the Distributions

In robust statistics the behaviour of estimators has commonly been studied on various particular parametric distributions, usually ones with long tails. The results have been extended to other distributions by assuming that it is the tail length of the distribution as measured by, say, kurtosis which determines the estimator to use, i.e. it is believed that if a good estimator is not known for a distribution one should use a good estimator for some distribution with a similar tail length.

But is this really true? Nobody has shown that if two distributions have the same kurtosis they admit similar good estimators. Are there other factors which play a large part in determining good estimators? (Is it really true that the ends justify the mean?)

In fact the double exponential distribution and the students-t distribution with 6 degrees of freedom have the same kurtosis but are best estimated differently; D'Agostino and Lee (1977) have shown that the asymptotically best linear combinations of order statistics for estimating the location parameters of the two distributions are quite different. Thus the use of the kurtosis seems less than ideal.

How can we improve on this approach?

It seems desirable that for a given set of data an estimator can be found that is good for all distributions likely to have produced the data. It would be desirable to use an estimator which is good for all distributions the shape of whose density function resembles the shape of the density of the data. The general shape of the density is more likely to be known than is its precise mathematical form.

Thus it seems useful to organize the set of distributions with respect to their shape and to map how the good estimators for distributions change as one moves gradually through this set. For example distributions could be classified according to the probability mass between zero and one standard deviation, between one and two standard deviations and so on. It could then be seen whether the good estimators for the distributions were roughly determined by their classification, and whether these estimators changed gradually as the class of the distribution changed gradually.

In this way a certain completeness may be attained in that any distribution could be fitted into the classification scheme and considered. Further it may be found that only certain features of a distribution's shape influence the form of its good estimators.

Before considering in more detail the choice of a classification scheme for the distributions an important property of the normal distribution must be established.

Definition 2.3.1

Let $F(x-\theta)$ be a distribution function depending on a location parameter θ and having variance σ^2, $0 < \sigma^2 < \infty$.

For sample size n and for an unbiased estimator, E_n, of θ let $V_{E_n} = \sup_\theta\{\text{variance of } E_n\}$ (provided this exists). The estimability of the distribution F is $\sigma^2 \Big/ \inf_{E_n} V_{E_n}$.

For simplicity assume that the location parameter θ is the population mean for the distribution $F(x-\theta)$. Then we have the following result.

Theorem 2.3.2

Let $n \geqslant 3$ and let X_j be independent random variables having the distribution function $F(x-\theta)$, $j=1,\ldots, n$, and let $\int x dF(x) = 0$ and $0 < \int x^2 dF(x) = \sigma^2 < \infty$ for all j. With respect to the estimation of the location parameter θ, no distribution is less estimable than the normal distribution.

Proof

For the normal distribution the mean is a uniformly minimum-variance unbiased estimator and has variance σ^2/n. Thus the estimability of the normal distribution is n.

However for any of the distributions the mean has variance σ^2/n. Thus their estimability must be $\geqslant n$. □

Furthermore the normal distribution is uniquely the least estimable distribution (for $n \geqslant 3$) in the sense that the location parameter of any other distribution whose best estimator can only guarantee a variance of σ^2/n over all values of θ, can be estimated more accurately than this for some θ (whereas for the normal distribution σ^2/n is uniformly the smallest variance that can be attained). This follows from the following theorem.

Theorem 2.3.3

Let $n \geqslant 3$ and let X_j be independent random variables having the distribution function $F(x-\theta)$, $j=1,\ldots, n$, and let $\int x dF(x) = 0$ and $0 < \int x^2 dF(x) = \sigma^2 < \infty$ for all j. The sample mean is admissible under quadratic loss, in the class of all unbiased estimators of θ if and only if the distribution function F is normal.

Proof

This result follows directly from Theorem 7.4.1 of Kagan, Linnik and Rao (1973). □

There is another sense in which the normal distribution is uniquely the least estimable distribution. Let $X_1, X_2, \ldots, X_n$ be the order statistics from a sample of size n from a distribution F(x) depending on a location parameter μ and a scale parameter σ only. Let $\sigma^2 \mathbf{B}$ be the covariance matrix of the order statistics. Lloyd (1952) has considered estimation of the location parameter using best (i.e. having minimum variance) unbiased linear combinations of order statistics. He showed that if the row sums of $\mathbf{B}$ are not all equal (to one) this best estimator has smaller sampling variance than the mean.

Govindarajulu (1966) and Bennett (1952) proved the following result:

Theorem 2.3.4

If F(x) is symmetric about zero then, for $i=1,\ldots, n$, $\sum_{j=1}^{n}$ covariance $(X_i, X_j) = 1$, $n=2,\ldots\ldots$, if and only if F(x) is the standard normal distribution function.

From these results can be deduced:

Theorem 2.3.5

If $F(x)$ is any nonnormal symmetric distribution depending on a location parameter μ and a scale parameter σ only, then for some sample size n there exists an estimator of μ with sampling variance less than σ^2/n.

Since the normal distribution is the least estimable distribution it can be regarded as a natural groundtstate. Thus in classifying distributions by their probability in various regions it seems natural to measure this relative to the probability which the normal distribution has in that region. In this way it could be seen how different deviations from normality can be used to achieve more accurate estimation than is possible for the normal distribution.

Theorem 2.3.6

If $f_1(x)$ and $f_2(x)$ are two distinct symmetric continuous density functions, each with mean μ and variance σ^2, they intersect at least twice on each side of μ.

Proof

Clearly they must intersect at least once on each side of μ.

Assume that they intersect exactly once on each side of μ - at $\pm K$. Assume $f_1(x) > f_2(x)$ between $\pm K$.

Let $g(x) = \min\{f_1(x), f_2(x)\}$.

Then

$$\sigma^2 = 2\int_0^\infty x^2 f_1(x)\,dx$$

$$= 2\int_0^\infty x^2 g(x)\,dx + 2\int_0^K x^2(f_1(x) - g(x))\,dx$$

Also

$$\sigma^2 = 2\int_0^\infty x^2 f_2(x)\,dx$$

$$= 2\int_0^\infty x^2 g(x)\,dx + \int_K^\infty x^2(f_2(x) - g(x))\,dx$$

$$\therefore \int_0^K x^2(f_1(x) - g(x))\,dx = \int_K^\infty x^2(f_2(x) - g(x))\,dx$$

But

$$\int_0^K x^2(f_1(x) - g(x))\,dx < K^2\int_0^K (f_1(x) - g(x))\,dx$$

and

$$\int_K^\infty x^2(f_2(x) - g(x))\,dx > K^2\int_K^\infty (f_2(x) - g(x))\,dx$$

$$= K^2\int_0^K (f_1(x) - g(x))\,dx$$

Thus

$$\int_0^K x^2(f_1(x) - g(x))\,dx < \int_K^\infty x^2(f_2(x) - g(x))\,dx.$$

This contradiction proves the theorem. □

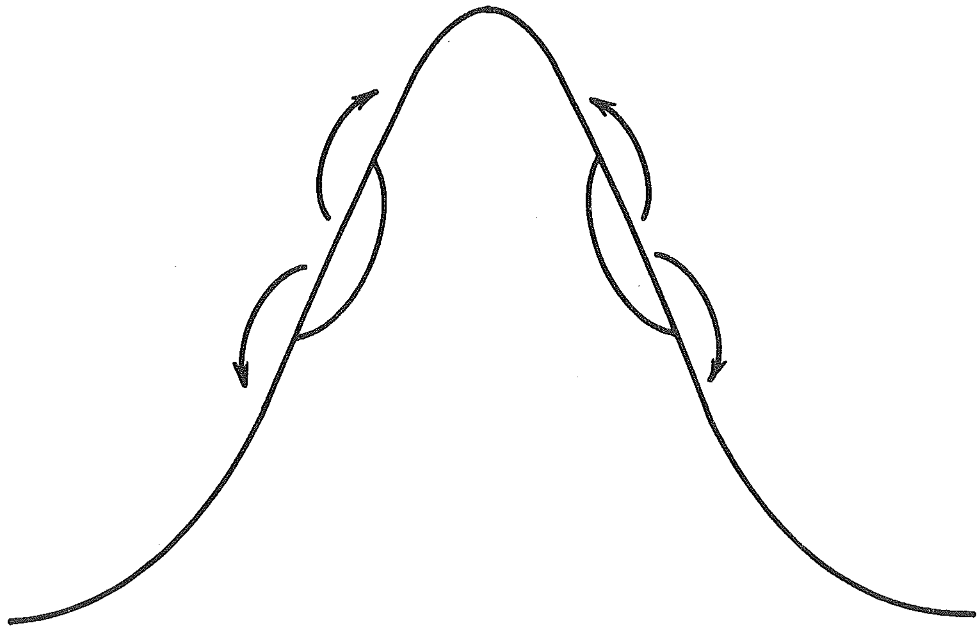
Thus any symmetric continuous density with mean zero and variance one cuts the standard normal density at least twice on each side of zero. In fact a few common densities which were checked cut the standard normal density exactly twice on each side of zero. Furthermore reasonably smooth densities which cut the normal density more than twice will, by and large, be closer to the normal density than those which cut it exactly twice.

Thus it seems natural to classify a distribution by giving the regions where its density is greater than the normal density and the regions where its density is less than the normal density and also, for each of these regions, the difference between the probability mass of the distribution and that of the normal distribution. Further, it seems reasonable to restrict attention, at least initially, to those symmetric distributions whose densities cut the normal density exactly twice on each side of the mean since this is both the most interesting case and the worst case.

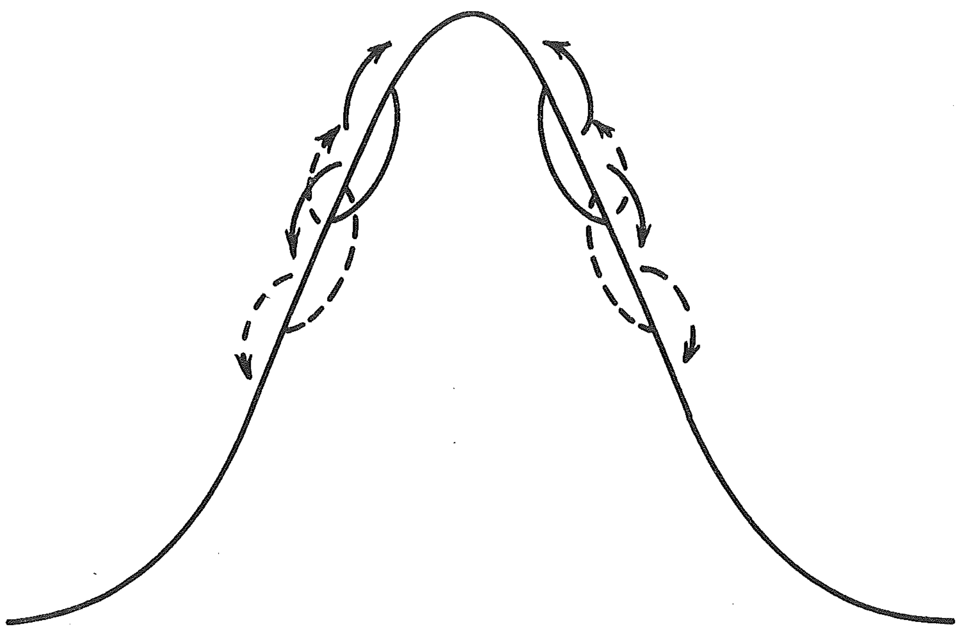
This classification can also be regarded as a refinement of kurtosis. A distribution with a high kurtosis can be obtained by moving some probability mass from the shoulder of the standard normal distribution and putting some of this further in and some further out in such a way that the variance is unchanged but the fourth moment is increased. (See Figure 2.3(a)). The same kurtosis could be obtained by moving different amounts of probability from different shoulder regions. (See Figure 2.3(b)). Thus if we specify the region from which the probability is moved and how much is moved we are measuring nonnormality in a more finely differentiated way than kurtosis does. The classification could be made more refined by increasing the number of regions in which the probability difference is specified. This could be increased until the density was quite closely tied down. However by keeping the number of regions as small as possible we are requiring less information to be assumed about the shape of the error distribution.

Hogg (see Hogg 1974) has developed adaptive estimators based on Q statistics such as:

$$Q = [\bar{U}(\cdot 05) - \bar{L}(\cdot 05)]/[\bar{U}(\cdot 5) - \bar{L}(\cdot 5)],$$



(a)



(b)

FIGURE 2·3

CHANGES IN THE NORMAL DISTRIBUTION WHICH INCREASE ITS KURTOSIS where $\bar{U}(\beta)$ is the average of the largest $n\beta$ order statistics and $\bar{L}(\beta)$ is the average of the smallest $n\beta$ order statistics. Perhaps the approach suggested above could be seen as being more in the spirit of Q statistics than of kurtosis inasmuch as consideration is given directly to how the probability mass is spread rather than to a high-order moment based on this.

The aim of this study was to test the usefulness of the above method of classifying distributions. The distributions considered were piecewise linear and were constructed with regard to their general shape, their classification with respect to the above scheme being varied systematically. "Best" estimators were to be found and it was hoped that these would change gradually as the distribution was changed gradually. It was also hoped that where standard distributions fitted into the scheme their "best" estimators would be similar to the estimators suggested by the classification scheme in the appropriate region. It was believed that in this way the known results would be generalized and made more systematic.

In particular it was hoped that the scheme would differentiate between the double exponential distribution and the students-t distribution with six degrees of freedom. Kurtosis fails to do this.

An advantage of the approach is that it allows parts of the density function's shape to be varied rather than requiring the choice of a density function as a whole. This allows more flexibility in finding a distribution to match given data. It may also allow a better understanding of the estimation process by breaking it into parts and may reveal what features of the shape of a density function are important in determining the estimator to use on it.

2.4 The Estimators

The estimators considered have been restricted to linear combinations of order statistics, hereafter called linear estimators. This was largely for computational convenience, the emphasis in the study being on the choice of the distributions. However I hope that general patterns which emerge will suggest similar patterns for other types of estimators.

An advantage of considering linear estimators is that for a given distribution it is possible to find linear estimators of location and scale which have minimum variance in the class of unbiased linear estimators. The method for finding these best linear estimators was developed independently by Lloyd (1952) and Bennett (1952).

In this study best linear estimators are found for the distributions described in the previous section.

2.5 The Calculations

Lloyd (1952) has shown that for a sample of size n from a symmetric distribution the best linear estimator of the mean is given by

$$\mu^* = \frac{\mathbf{1}'\Omega\mathbf{X}}{\mathbf{1}'\Omega\mathbf{1}}$$

and its variance by

$$\text{var } \mu^* = \frac{\sigma^2}{\mathbf{1}'\Omega\mathbf{1}}$$

where σ is a scale parameter,

$\Omega = \mathbf{B}^{-1}$, $\sigma^2\mathbf{B}$ being the covariance matrix of the order statistics,

$\mathbf{X}$ is the vector of order statistics,

$\mathbf{1}$ is a column vector of n 1's.

The calculation of the best linear estimator for a distribution thus requires the calculation of the covariance matrix of the order statistics for that distribution. To calculate the covariances for the piecewise linear distributions with finite range considered here, the region of integration was broken into triangles and rectangles and Gaussian quadrature formulae for such regions were used. The calculations are exact apart from roundoff error.

Sample sizes 4, 8 and 16 were considered and details of the distributions and results are given in the appendices. Analysis of the results will concentrate on sample size 8.

The distributions considered were non-zero on only a finite range. It was believed that for small sample sizes the results would not be much different from the results for distributions which are similar except for having infinite tails. To test this, approximations having finite range were made to several standard distributions and the results compared to the known results for these distributions.

For example the standard normal density was approximated by a density which was non-zero between ±4·68. The best linear estimator was calculated for sample sizes, N, of 8 and 16.

Table 2.5.1 gives the equations of the straight line segments which make up the distribution. The R^{th} segment extends from P [R] to P [R + 1] and has equation Y = A [R] X + B [R].

Tables 2.5.2 and 2.5.3 give the expected values and covariances of the order statistics along with the corresponding expected values and covariances for the normal distribution taken from Teichroew (1962).

TABLE 2.5.1

Parameters of an Approximation to the Standard Normal Density

R	P |R|	A |R|	B |R|
1	−4·68363603266	0·000135639704009	0·00063528700507
2	−3·99939598088	0·00141041327633	0·0056908212723
3	−3·5001785904	0·066886011956	0·0246146369684
4	−2·99954698591	0·0256296321525	0·080199086531
5	−2·50032959544	0·07887357071	0·211030181828
6	−1·90070304026	0·213558391022	0·456036224121
7	−0·35001785904	0·072550753768	0·403640943487
8	0		

TABLE 2.5.2

Expected Values of Order Statistics from an Approximation to the Normal Distribution and from the Normal Distribution

I	Approximation, N = 8	Normal, N = 8	Approximation, N = 16	Normal, N = 16
1	−1·4227	−1·4236	−1·7631	−1·7660
2	−0·8539	−0·8522	−1·2847	−1·2847
3	−0·4742	−0·4728	−0·9928	−0·9902
4	−0·1529	−0·1525	−0·7660	−0·7632
5			−0·5719	−0·5700
6			−0·3972	−0·3962
7			−0·2341	−0·2338
8			−0·0774	−0·0773

Covariances of Order Statistics from an Approximation to the Normal Distribution and from the Normal Distribution

I	J	Approximation, N=8	Normal, N=8	Approximation, N=16	Normal, N=16
1	1	0.3696	0.3729	0.2937	0.2950
1	2	0.1850	0.1863	0.1429	0.1449
1	3	0.1257	0.1260	0.0975	0.0985
1	4	0.0945	0.0947	0.0752	0.0754
1	5	0.0746	0.0748	0.0613	0.0613
1	6	0.0601	0.0602	0.0516	0.0516
1	7	0.0481	0.0483	0.0444	0.0446
1	8	0.0365	0.0368	0.0388	0.0390
1	9			0.0344	0.0345
1	10			0.0307	0.0308
1	11			0.0275	0.0275
1	12			0.0246	0.0246
1	13			0.0220	0.0220
1	14			0.0193	0.0195
1	15			0.0166	0.0169
1	16			0.0137	0.0138
2	2	0.2385	0.2394	0.1710	0.1744
2	3	0.1635	0.1632	0.1173	0.1191
2	4	0.1236	0.1233	0.0908	0.0914
2	5	0.0978	0.0976	0.0742	0.0745
2	6	0.0790	0.0787	0.0625	0.0628
2	7	0.0632	0.0632	0.0538	0.0542
2	8			0.0471	0.0475
2	9			0.0417	0.0421
2	10			0.0372	0.0375
2	11			0.0334	0.0336
2	12			0.0299	0.0300
2	13			0.0267	0.0268
2	14			0.0235	0.0237
2	15			0.0202	0.0206
3	3	0.2020	0.2008	0.1351	0.1363
3	4	0.1533	0.1524	0.1048	0.1049
3	5	0.1217	0.1210	0.0857	0.0855
3	6	0.0984	0.0978	0.0723	0.0722
3	7			0.0623	0.0624
3	8			0.0546	0.0547
3	9			0.0484	0.0484
3	10			0.0432	0.0431
3	11			0.0387	0.0387
3	12			0.0348	0.0346
3	13			0.0310	0.0309
3	14			0.0272	0.0274
4	4	0.1883	0.1872	0.1186	0.1179
4	5	0.1500	0.1492	0.0972	0.0962
4	6			0.0821	0.0813
4	7			0.0708	0.0703
4	8			0.0620	0.0617
4	9			0.0549	0.0547
4	10			0.0491	0.0488

TABLE 2.5.3 (continued)

I	J	Approximation, N=8	Normal, N=8	Approximation, N=16	Normal, N=16
4	11			0·0440	0·0437
4	12			0·0395	0·0391
4	13			0·0352	0·0349
5	5			0·1086	0·1074
5	6			0·0918	0·0908
5	7			0·0792	0·0785
5	8			0·0694	0·0689
5	9			0·0616	0·0611
5	10			0·0550	0·0546
5	11			0·0494	0·0489
5	12			0·0443	0·0438
6	6			0·1020	0·1010
6	7			0·0881	0·0875
6	8			0·0773	0·0768
6	9			0·0685	0·0682
6	10			0·0613	0·0609
6	11			0·0550	0·0545
7	7			0·0979	0·0974
7	8			0·0859	0·0856
7	9			0·0763	0·0760
7	10			0·0682	0·0679
8	8			0·0959	0·0957
8	9			0·0852	0·0850

TABLE 2.5.4

Coefficients and Variances of Best Linear Estimators of Location for an Approximation to the Normal Distribution

I	N = 8	N = 16
1	0·1277	0·0611
2	0·1279	0·0705
3	0·1199	0·0678
4	0·1244	0·0565
5		0·0548
6		0·0596
7		0·0640
8		0·0659
Variance	0·1250	0·0625

Table 2.5.4 gives the coefficients of the best linear estimators.

In each table values omitted may be found from considerations of symmetry.

The percentage difference from the coefficients of the sample mean is always less than 4·1% for N = 8 and 12·8% for N = 16. Approximations to other distributions yielded similar results. The agreement seems close enough to support the use of piecewise linear distributions in seeking overall trends in the way the best linear estimator varies as the shape of the distribution varies.

The close agreement of the variance of the estimator with that of the mean on the normal distribution is encouraging. This suggests some robustness in the sense that small changes in the coefficients of the estimator and in the distribution do not much affect the variance of the estimator.

2.6 Analysis of the Results for N = 8

Forty-seven distributions were constructed, almost all having high tails. Each had variance one and was classified, firstly, according to where its density was above, and where below, the standard normal density. To this end two "cutpoints" are given. For the high-tailed distributions the region between the two cutpoints, and the corresponding region on the other side of the mean, are the regions where the density is less than the normal density. (For the low-tailed distributions these are the regions where the density is *greater* than that of the normal density). Of course, since all the distributions are non-zero on only a finite range, even the high-tailed distributions have densities less than the normal density in the *extreme* tails but this factor was considered negligible for small sample sizes.

Table 2.6.1 shows the cutpoints for the 47 distributions. For given values (in standard deviations) of the left cutpoint on the right-hand side of the distribution, and the distance apart of the two cutpoints, the number of distributions considered is given. The positions at which various standard distributions fit in are also given. Note that all values given for cutpoints are approximate.

Let us now examine some of the results for N = 8 for long-tailed distributions in the four series in which the cutpoints are distance 2 apart. A summary of these results is given in Tables 2.6.2, 2.6.3, 2.6.4 and 2.6.5. The full results are given in the appendices.

In any one series the distributions have the same cutpoints. Within the series they are classified according to the difference between the probability mass of the distribution and that of the normal distribution between the cutpoints. It was decided to concentrate on these three parameters (the two cutpoints and this probability difference) in classifying distribution shapes as it was found in practice that these three parameters went a good way towards tying down the approximate shape of the density function. In each series the first distribution has the maximum probability difference possible for a symmetric density each of whose halves is monotonic and which has the given cutpoints. Within each series probability differences in the order of 0·16, 0·08, 0·04 and 0·02 have been among those considered.

TABLE 2.6.1

The Number of Distributions Considered for Each Pair of Cut-points and the Cutpoints for Various Standard Distributions.

	Separation of the cutpoints					
Left Cutpoint	1·0	1·25	1·5	1·75	2	2·25
0·25				3	6	2
0·5					9 D.E.	
0·75	2			8 S.t.	6 10%CN	3
1·0					4 5% CN	
1·25				2		
1·5	2					

The categories into which various standard distributions most nearly fit (after being scaled to have variance one) are indicated. S.t. refers to the students-t distribution with 6 degrees of freedom, D.E. to the double exponential distribution, 10% CN to the standard normal distribution N(0,1) with 10% contamination from N(0,9) and 5% CN to N(0,1) with 5% contamination from N(0,9).

TABLE 2.6.2

The Coefficients W[I] and Variances of the Best Linear Estimators for N = 8 for Some Distributions Having Cutpoints 0·25 and 2·25.

Probability Difference	W[1]	W[2]	W[3]	W[4]	Variance of Estimator
0·3256	-0·0071	-0·0188	-0·0193	0·5452	0·0069
0·1648	-0·0114	-0·0412	0·0027	0·5500	0·0314
0·0842	-0·0021	-0·0013	0·0897	0·4137	0·0808
0·0667	0·0103	0·0235	0·1059	0·3603	0·0953
0·0371	0·0544	0·0694	0·1190	0·2572	0·1162
0·0198	0·0935	0·0949	0·1260	0·1855	0·1232

TABLE 2.6.3

The Coefficients W[I] and Variances of the Best Linear Estimators for N = 8 for Some Distributions Having Cutpoints 0·5 and 2·5.

Probability Difference	W[1]	W[2]	W[3]	W[4]	Variance of Estimator
0·2673	-0·0124	-0·0190	0·1439	0·3876	0·0161
0·1570	-0·0182	-0·0273	0·1297	0·4157	0·0430
0·0924	-0·0135	0·0124	0·1495	0·3516	0·0770
0·0368	0·0265	0·0926	0·1526	0·2283	0·1126
0·0208	0·0654	0·1104	0·1379	0·1863	0·1212

TABLE 2.6.4

The Coefficients W[I] and Variances of the Best Linear Estimators for N = 8 for Some Distributions Having Cutpoints 0·75 and 2·75.

Probability Difference	W[1]	W[2]	W[3]	W[4]	Variance of Estimator
0·2055	-0·0163	0·0362	0·3006	0·1796	0·0319
0·1679	-0·0198	0·0417	0·2910	0·1871	0·0420
0·1091	-0·0210	0·0661	0·2359	0·2190	0·0617
0·0795	-0·0188	0·0908	0·2083	0·2197	0·0785
0·0410	0·0004	0·1290	0·1595	0·2111	0·1039
0·0172	0·0502	0·1509	0·1444	0·1545	0·1206

TABLE 2.6.5

The Coefficients W[I] and Variances of the Best Linear Estimators for N = 8 for Some Distributions Having Cutpoints 1 and 3.

Probability Difference	W[1]	W[2]	W[3]	W[4]	Variance of Estimator
0·1484	-0·0190	0·1168	0·2860	0·1162	0·0538
0·0861	-0·0203	0·1659	0·2248	0·1296	0·0763
0·0473	-0·0066	0·1854	0·1853	0·1358	0·1020
0·0263	0·0194	0·2087	0·1516	0·1202	0·1141

On the whole the best estimators change reasonably smoothly both within a given series and from series to series. Within each series the estimator moves gradually towards the mean as the probability difference decreases. In moving from series to series as the envelope between the cutpoints is gradually moved outwards, the weight in the best estimator gradually moves outwards.

Consider the series with cutpoints 0·25 and 2·25. The best linear estimator for the most nonnormal distribution (i.e. the distribution with maximum probability difference) is very heavily weighted to the middle, with all coefficients other than the middle pair being negative. As the probability difference is reduced the weight moves out from the centre and, with one small exception, the coefficients move monotonicly towards 0·125.

Consider the series with cutpoints 0·5 and 2·5. Here the best estimator for the most nonnormal distribution is not quite so heavily weighted towards the middle, with the second pair from the middle now receiving significant weight. Again, as the probability difference is reduced, the weight moves out towards the extremes. Some coefficients do not change monotonicly towards 0·125 but deviations from this pattern are small. Some such deviations are not surprising. For example the increase in W[3] away from 0·125 could be considered as being caused by the rapid movement of weight outward from the very middle pair of order statistics in order to bring these middle coefficients closer to 0·125.

Consider the series with cutpoints 0·75 and 2·75. By this stage the weight has moved outward to the extent that, for the most nonnormal distribution, the central pair of order statistics no longer receive the maximum weight. Again as the distribution becomes closer to the normal distribution the estimator becomes closer to the mean, the weight moving out to the extremes.

Consider the series with cutpoints 1 and 3. The weight for the most nonnormal distribution has moved a little further out. Again the general trend within the series is for the weight to move out and for the estimator to get closer to the mean as the distribution becomes more like the normal distribution.

Thus we see that there is an overall trend for weight to move outwards from the centre as the cutpoints move outwards. There is also a trend for the weight to move outwards as the distribution becomes more like the normal distribution, so that the estimators become more like the mean. Despite this trend individual coefficients do not always move monotonicly towards 0·125. Often the very middle coefficient W[4] moves slightly away from 0·125 before moving towards it. Further it often happens that as the weight moves out to the extremes it is not evenly distributed at first so that W[2], for example, increases beyond 0·125 while W[1] still carries little weight.

However too much significance should not be attached to small changes in the weights because these may be affected somewhat by the particular distribution chosen with the given cutpoints and probability difference. Rather the results should be used to show overall trends in the best estimator as the shape of the distribution changes.

It should be noted that similar patterns exist for the other series of distributions considered (as can be seen from an examination of the appendices).

Two pairs of similar distributions ((4,4) and (4,5), (7,3) and (7,4)) were considered in order to see whether small changes in the distribution made much difference to the best estimator, and thus to further test the continuity or the pattern in the results as the distribution is changed. Examination of the results in the appendices shows that the changes made very little difference. The main difference between the distributions in each pair is that one is non-zero until further out than the other. That this made little difference to the results suggests that the use of distributions which are non-zero on only a finite range does not affect the results seriously.

We now consider two distributions for which the best linear estimators for $N = 8$ are known and see how closely these estimators agree with the estimators for the categories into which these distributions most nearly fit.

The normal distribution $N(0,1)$ contaminated with 10% $N(0,9)$ and scaled to have variance 1 (i.e. $0{\cdot}9\ N(0, \frac{1}{1{\cdot}8}) + 0{\cdot}1\ N(0, \frac{9}{1{\cdot}8})$) has cutpoints 0·785 and 2·76 and probability difference 0·056 (approximately). The series into which it most nearly fits is the one with cutpoints 0·75 and 2·75. Its probability difference falls between the 0·0795 and 0·0410 values considered in that series. Table 2.6.6 shows the best linear estimators and their variances for these distributions along with those of the 10% contaminated normal distribution itself (taken from Gastwirth and Cohen (1970), the variance given there being suitably scaled).

TABLE 2.6.6

The Coefficients W[I] and Variances of the Best Linear Estimators for N = 8 for the 10% Contaminated Normal Distribution and two Nearby Distributions with Cutpoints 0·75 and 2·75.

	Probability Difference	W[1]	W[2]	W[3]	W[4]	Variance of Estimator
	0·041	0·0004	0·1290	0·1595	0·2111	0·1039
(10%CN)	0·056	-0·0120	0·1275	0·2016	0·1829	0·0957
	0·079	-0·0188	0·0908	0·2083	0·2197	0·0785

TABLE 2.6.7

The Coefficients W[I] and Variances of the Best Linear Estimators for N = 8 for the 5% Contaminated Normal Distribution and two Nearby Distributions with Cutpoints 1 and 3.

	Probability Difference	W[1]	W[2]	W[3]	W[4]	Variance of Estimator
	0·026	0·0194	0·2087	0·1516	0·1202	0·1141
(5%CN)	0·035	0·0050	0·1708	0·1690	0·1551	0·1080
	0·047	-0·0066	0·1854	0·1853	0·1358	0·1020

The agreement between the 10% CN case and the other two distributions seems reasonably good, three of the four coefficients and the variance for the 10% CN case being between the corresponding values for the other two distributions.

The normal distribution N(0,1) contaminated with 5% N(0,3) (i.e. $0{\cdot}95\ N(0, \frac{1}{1{\cdot}4}) + 0{\cdot}05\ N(0, \frac{9}{1{\cdot}4})$) has cutpoints 0·845 and 2·96 and probability difference 0·035 (approximately). Of the fuller series the one into which it most nearly fits is the one with cutpoints 1 and 3. Its probability difference falls between the 0·0263 and 0·0473 values considered in that series. Table 2.6.7 shows the best linear estimators and their variances for these distributions along with those of the 5% contaminated normal distribution itself. Again the agreement between the 5% CN case and the other two distributions seems reasonably good.

Thus, from the somewhat limited evidence considered, it seems that results from standard distributions are reasonably similar to those obtained from the classification scheme. Further, as we saw earlier, the results from the classification scheme vary reasonably smoothly and cover a wide range of distributions. Thus it seems that the classification scheme provides a generalization of known results.

<u>The Double Exponential Distribution and the Students-t Distribution with Six Degrees of Freedom</u>.

One of the main aims of this project was to devise a system for classifying distributions which would distinguish between the double exponential distribution and the students-t distribution with 6 degrees of freedom. Kurtosis fails to do this: both distributions have the same kurtosis although the asymptotically best linear estimators for the two distributions are quite different, with the estimator for the double exponential distribution being more heavily weighted towards the centre than that for the students-t distribution.

The cutpoints for the double exponential distribution (scaled to have variance one) are 0·485 and 2·34 and the probability difference is 0·071 (approximately). The nearest distribution to this in the system is that with cutpoints 0·5 and 2·5 and probability difference 0·092. The cutpoints for the students-t distribution with 6 degrees of freedom (scaled to have variance one) are 0·705 and 2·51 and the probability difference is 0·035 (approximately). The nearest distribution to this in the system is that with cutpoints 0·75 and 2·5 and probability difference 0·041. Table 2.6.8 shows the best linear estimators and their variances for these two distributions.

The two estimators are significantly different. Furthermore the estimator for the distribution near the double exponential distribution has significantly more weight in the middle than does the estimator for the distribution like the students-t distribution. Thus it seems that the classification system, unlike the kurtosis, is successful in distinguishing between the 2 distributions.

It is worth noting that a piecewise linear approximation was made to the double exponential distribution directly. The coefficients of the best linear estimator for N = 8 were given by:

$W[1] = 0{\cdot}0011 \quad W[2] = 0{\cdot}0210 \quad W[3] = 0{\cdot}1328 \quad W[4] = 0{\cdot}3451.$

These coefficients are fairly close to those given in Table

2.6.8 for the distribution near the double exponential distribution in the classification system.

A Simplification of the Classification Scheme

It seems that a reasonable idea of the estimator to use on a distribution can be obtained using the two cutpoints and the probability difference. Is it possible that a good approximation to the estimator can be obtained if, instead of using both cutpoints, we use only their midpoint (i.e. of the cutpoints on one side of the distribution) and ignore how far apart they are? i.e. is it possible that the estimator depends on what the probability difference is and where it is centred and not much on how spread out it is?

To test this, four pairs of distributions were considered. For the two distributions in each pair, the midpoints of the two cutpoints on the right (left) side of the distribution are the same and the probability differences are similar but one of the distributions has its cutpoints distance 1 apart and the other has them distance 2 apart. Table 2.6.9 shows the best linear estimators and their variances for these distributions.

In each pair the best linear estimators and their variances are fairly similar. Thus it seems that a good idea of the best estimator to use can be obtained using only the midpoint of the cutpoints and the probability difference. It would be worthwhile to test this hypothesis over a wider range of distributions.

Table 2.6.10 shows the best estimators for the distributions organized with respect to these two parameters. Examination of the table should reveal how much the two

TABLE 2.6.8

The Coefficients W[I] and Variances of the Best Linear Estimators for N = 8 for a Distribution Near the Double Exponential Distribution and a Distribution Near the Students-t Distribution with 6 Degrees of Freedom.

Cutpoints	Probability Difference	W[1]	W[2]	W[3]	W[4]	Variance of Estimator
0·5 ,2·5	0·092	-0·0135	0·0124	0·1495	0·3516	0·0770
0·75,2·5	0·041	0·0108	0·1203	0·1797	0·1892	0·1098

Note that the first distribution is close to the double exponential distribution and the second to the students-t distribution with 6 degrees of freedom.

TABLE 2.6.9

The Coefficients W[I] and Variances of the Best Linear Estimators for N = 8 for 4 Pairs of Distributions Where in Each Pair the 2 Distributions Have Cutpoints Centred About the Same Point and Have Similar Probability Differences.

Cutpoints	Probability Difference	W[1]	W[2]	W[3]	W[4]	Variance of Estimator
0·75,1·75	0·100	0·0075	-0·0181	0·1444	0·3662	0·0876
0·25,2·25	0·084	-0·0021	-0·0013	0·0897	0·4137	0·0808
0·75,1·75	0·021	0·0978	0·0816	0·1460	0·1746	0·1232
0·25,2·25	0·020	0·0935	0·0949	0·1260	0·1855	0·1232
1·5 ,2·5	0·043	0·0059	0·2247	0·1692	0·1003	0·1110
1 , 3	0·047	-0·0066	0·1854	0·1853	0·1358	0·1020
1·5 ,2·5	0·021	0·0442	0·1912	0·1430	0·1217	0·1204
1 , 3	0·026	0·0194	0·2087	0·1516	0·1202	0·1141

Table 2 .6.10

The Coefficients W[1], W[2], W[3], W[4] of
the Best Linear Estimator for Sample Size Eight for
Forty-seven Distributions

Probability Difference	The Midpoint of the Cutpoints 1·125	1·25	1·375	1·5	1·625	1·75	1·875	2·0	2·125
0·355			−·01,−·01,·01,·51						
0·325		−·01,−·02,−·02,·55							
0·285	−·01,−·03,−·03,·56								
0·265				−·01,−·02,·14,·39 & −·02,−·02,·18,·36					
0·215							−·01,·05,·31,·15		
0·21						−·02,·04,·30,·18			
0·20									
0·19					−·02,·00,·26,·26				
0·18									
0·17						−·02,·04,·29,·19			
0·16		−·01,−·04,·00,·55		−·02,−·03,·13,·42 &	−·02,·01,·26,·25				
0·15				−·02,−·03,·12,·42				−·02,·12,·29,·12	
0·14									
0·13									
0·12									
0·11						−·02,·07,·24,·22			
0·10		·01,−·02,·14,·37							−·01,·27,·16,·08
0·09				−·01,·01,·15,·35					
0·08		−·00,−·00,·09,·41			−·02,·06,·22,·24	−·02,·09,·21,·22		−·02,·17,·22,·13	
0·07		·01,·02,·11,·36							
0·06					−·00,·10,·20,·20 &				
0·05					−·00,·10,·20,·20			·01,·22,·17,·10 &	
0·04					·01,·12,·18,·19	·00,·13,·16,·21		−·01,·19,·19,·14	
0·03		·05,·07,·12,·26		·03,·09,·15,·23	·04,·13,·16,·17			·02,·21,·15,·12	
0·02	·08,·10,·13,·19	·09,·09,·13,·19 &	·09,·10,·12,·19	·07,·11,·14,·19			·04,·16,·15,·16	·04,·19,·14,·12	·03,·20,·15,·12
0·01	·10,·10,·13,·18	·10,·08,·15,·17				·05,·15,·14,·15	·05,·16,·14,·15		
0·00									
−0·01				·18,·11,·11,·10	·18,·11,·11,·10				
−0·02				·21,·11,·10,·07					

parameters determine the best linear estimator for a distribution.

The most obvious patterns are that weight moves out from the centre as one moves down the table or to the right of the table. At the top left of the table the estimators are very heavily weighted towards the centre. We see that W[4] tends to decrease going down a column and going to the right. Far enough to the right, weight has moved from W[4] to the extent that it is no longer the biggest coefficient. Thus going down these columns W[4] may increase at first, as weight from the bigger coefficients further out is distributed more evenly. Going down these columns we find a fairly smooth decrease in whichever coefficient is biggest at the top of the column.

Why does weight move out from the centre as one moves down or to the right of the table? Perhaps the reason is that, as we shall consider further in Chapter 4, the higher the probability density is in a region, the more weight should go to observations likely to fall there. Thus when the probability difference is large, and consequently the probability density is high in the middle, weight is given to the middle observations. Further the narrower this central region of high density is, the further the weight must go to the middle in order to take advantage of it.

It is worth noting how useful the "second" variable, the midpoint of the cutpoints, is in determining the best estimator. The changes in the best estimator going across the table are regular and quite strong. The usefulness of the probability difference is also clear - for example the estimators for distributions with small probability differences are much closer to the mean than those for distributions with big probability differences.

Overall the best estimator changes reasonably smoothly as one moves through the table so that the position of a distribution in the table approximately determines its best estimator.

Estimability

We saw earlier that the normal distribution is the least estimable distribution. Examination of the results in the appendices shows that, further, the more nonnormal a distribution is the more estimable it is: within any one series of high-tailed distributions the greater the probability difference the smaller the variance of the best linear estimator.

This pattern is also shown in Table 2.6.11 which shows the variances of the best estimators at different places within the classification scheme. The variance of the best linear estimator changes reasonably smoothly as the classification of the distribution changes. In fact the variance of the best linear estimator is fairly well determined by the probability difference, which serves as a rough measure of non-normality. The main exception is the second of the pair of distributions with probability difference between 0·26 and 0·27. It should be noted that this is an unusual distribution whose density is made of three horizontal straight lines.

Table 3.1.1 shows which distributions the figures in Table 2.6.11 refer to.

Low-tailed Distributions

Unfortunately time did not permit a similar systematic

TABLE 2.6.11

THE VARIANCE OF THE BEST LINEAR ESTIMATOR FOR FORTY-EIGHT DISTRIBUTIONS

CENTRE OF CUTPOINTS

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.004						
0.35										
0.34										
0.33										
0.32			0.007							
0.31										
0.3										
0.29										
0.28		0.015								
0.27										
0.26					0.016& 0.028					
0.25										
0.24										
0.23										
0.22								0.029		
0.21							0.032			
0.2										
0.19						0.039				
0.18										
0.17							0.042			
0.16			0.031			0.045				
0.15					0.043& 0.044				0.054	
0.14										
0.13										
0.12										
0.11							0.062			
0.1			0.088							0.075
0.09			0.081		0.077	0.076			0.076	
0.08							0.079			
0.07			0.095							
0.06					0.101& 0.103					
0.05									0.111& 0.102	
0.04			0.116		0.113	0.110	0.104			
0.03			[0.123			0.119			0.114	
0.02		0.122[	&	0.123	0.121			0.119	0.120	0.119
0.01		0.124[	0.123				0.121	0.121		
0.00	0.125									
-.01					0.124	0.123				
-.02					0.121					

study of low-tailed distributions. However examination of the results for the three that were studied (Distributions (5,8),(8,8) and (8,9)) suggests that for low-tailed distributions extra weight should be placed on the extreme observations and less weight on the central observations.

Conclusion

It seems that there is enough of a pattern in the results to suggest that the general idea of classifying distributions by their shape or the approximate position of the probability mass, is worth pursuing. The results require further testing with more distributions - especially ones with infinite tails. They should also be tested on other sample sizes. If necessary distributions could be classified more precisely by increasing the number of regions in which the probability is specified.

CHAPTER 3

THE ROBUSTNESS OF LINEAR ESTIMATORS ON NEAR-NORMAL DENSITIES OF VARIOUS SHAPES

3.1 Introduction

An estimator is considered robust if its performance is good not only for a given error distribution but also for distributions which are similar to the given one. In Chapter 2 an attempt was made to organize the set of distributions systematically and to find a set of features of a distribution which are useful for determining a good estimator to use on it. In this chapter consideration is given to the robustness of linear estimators over sets of distributions organized in this way.

Emphasis is placed on the choice of a measure of quality. An estimator's quality is measured firstly by its efficiency relative to the best linear estimator and secondly by its variance when each distribution is scaled to have variance one. The results obtained using these two measures are compared and a third measure of quality is suggested as being better. Using this measure an assessment of the robustness of the estimators is made and, in the light of this, the value of efficiency as a measure is reconsidered.

The distributions considered are the same ones as were considered in Chapter 2 including the approximation to the normal distribution given in Section 2.5. Results are tabulated in Appendix 5 and are given within the chapter in tables structured according to the parameters suggested in

Section 2.6 for describing the shape of a density (i.e. the probability difference between the cutpoints on one side and the midpoint of the cutpoints on one side). Table 3.1.1 shows which distributions occupy which places in these tables.

3.2 Robustness of Efficiency

The robustness of efficiency of linear estimators will now be considered. Efficiency is defined as (variance of best available estimator/variance of estimator) and in this section the best available estimator for a distribution is taken to be the best linear estimator. The use of efficiency as a measure is common in robustness studies.

Forty-one estimators were considered for sample size eight but the results are given only for some of these (including the best ones). In considering the robustness of an estimator emphasis should be placed on distributions in the lower group in the tables (i.e. with probability differences less than about .11) as the other distributions are probably unrealistic, being extremely nonnormal. However these extreme distributions are interesting in that they help to reveal overall trends.

The estimators discussed have weights (W[1], W[2],..., W[8]) with W[I] = W[9 - I] and with (W[1], W[2], W[3], W[4]) in the ratios: (0,0,0,1), (0,0,1,2), (0,0,1,1), (0,1,2,2), (0,1,1,1), (1,1,2,4), (1,1,2,2), (1,1,1,1), (-1,2,15,10) and (-1,2,10,15). The weights themselves can be found using the fact that the weights for each estimator sum to one. Among the estimators considered were approximations to the best linear estimators found in Chapter 2. The results are given

TABLE 3.1.1

THE DISTRIBUTIONS WHOSE RESULTS ARE GIVEN AT THE VARIOUS POSITIONS IN THE TABLES

CENTRE OF CUTPOINTS

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				(10,1)						
0.35										
0.34										
0.33										
0.32			(6,1)							
0.31										
0.3										
0.29										
0.28		(3,1)								
0.27										
0.26					(7,1) & (7,2)					
0.25										
0.24										
0.23										
0.22										
0.21								(11,1)		
0.2							(8,1)			
0.19						(4,1)				
0.18										
0.17							(8,2)			
0.16			(6,2)			(4,2)				
0.15					(7,3) & (7,4)				(9,1)	
0.14										
0.13										
0.12										
0.11							(8,3)			
0.1			(1,1)							
0.09					(7,5)					(5,1)
0.08			(6,3)			(4,3)	(8,4)		(9,2)	
0.07										
0.06			(6,4)							
0.05						(4,4) & (4,5)				
0.04						(4,6)	(8,5)		(2,1) & (9,3)	
0.03			(6,5)		(7,6)					
0.02		(3,2) (3,3)	(6,6) & (1,2)	(10,2)	(7,7)	(4,7)		(11,2) (11,3)	(9,4) (2,2)	(5,2)
0.01							(8,6)			
0.00	(12,1)									
-.01					(7,8)					
-.02					(7,9)	(4,8)				

in Tables 3.2.1 to 3.2.10 and are tabulated in Appendix 5.

The median is perhaps the most robust estimator over the whole set of distributions being the only one to provide 90 percent efficiency on the most nonnormal distributions. However other estimators are better for the more realistic near-normal distributions. The trends in the performance of the median as the distribution is changed, are interesting. Its efficiency decreases from over 90 percent for the most non-normal distributions down to less than 80 percent for distributions only slightly higher-tailed than normal, and still less for the normal distribution and lower-tailed distributions. The efficiency decreases as the midpoint of the cutpoints moves out. This is to be expected since the narrower the central high-probability region is, the more the weight must be put on the central observations in order to take advantage of it. (This sort of argument is discussed more in Chapter 4.) In fact when the midpoint of the cutpoints is very close to the centre of the distribution then for high probability differences the best linear estimators are even more heavily weighted towards the middle than the median is (see Table 2.6.10) so that initially the efficiency of the median increases with decreasing probability difference.

The estimator with weight ratios (0,0,1,2) is similar to the median but achieves its best performance for slightly lower probability differences,being worse for high probability differences and better for low probability differences. The doubly-trimmed mean, giving still less weight to the centre, carries this trend further and achieves its best performance for still lower probability differences. It seems to be best

TABLE 3. 2. 1

THE EFFICIENCY, ON FORTY-EIGHT DISTRIBUTIONS, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 0, 0, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				92.033						
0.35										
0.34										
0.33			90.161							
0.32										
0.31										
0.3										
0.29		91.418								
0.28										
0.27				92.587&91.087						
0.26										
0.25										
0.24										
0.23										
0.22								83.802		
0.21							85.554			
0.2										
0.19						89.729				
0.18										
0.17			95.013				86.452			
0.16				96.156&96.164		90.209				
0.15									80.411	
0.14										
0.13										
0.12										
0.11							89.376			
0.1			97.412							72.439
0.09			98.893		96.860	90.957			81.168	
0.08							89.570			
0.07			97.052							
0.06					88.394&88.626					
0.05								76.243&81.269		
0.04			89.618		89.034	86.607	88.011			
0.03						83.339			77.964	
0.02		83.268 81.300	[82.138 & [81.737	82.069	83.611		80.958	81.525 80.337	77.221	77.021
0.01										
0.00	73.895									
-.01					68.422	67.749				
-.02					63.210					

TABLE 3. 2. 2

THE EFFICIENCY,ON FORTY-EIGHT DISTRIBUTIONS,OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 0, 1, 2)

P.DIF	CENTRE OF CUTPOINTS 0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				71.429						
0.35										
0.34										
0.33										
			62.901							
0.32										
0.31										
0.3										
0.29										
		65.237								
0.28										
0.27										
				93.343&94.630						
0.26										
0.25										
0.24										
0.23										
0.22										
								92.911		
0.21										
							94.132			
0.2										
0.19						96.152				
0.18										
0.17							94.695			
			83.683							
0.16						96.626				
				5.496&95.071						
0.15									89.922	
0.14										
0.13										
0.12										
0.11							96.498			
0.1			99.738							
										81.519
0.09					99.652					
			99.033			97.650			90.122	
0.08							96.615			
0.07										
			99.496							
0.06										
					95.884&96.089					
0.05										
								85.339&89.967		
0.04						94.268	95.048			
			95.126		95.588					
0.03										
			[89.117			91.148			86.627	
0.02		90.102	&	88.919	90.816			89.463	85.703	85.674
		88.383	[89.070				88.892	88.290		
0.01										
0.00	81.709									
-.01					76.241					
						75.445				
- 02					70.997					

TABLE 3. 2. 3

THE EFFICIENCY,ON FORTY-EIGHT DISTRIBUTIONS,OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 0, 1, 1)

P.DIF	CENTRE OF CUTPOINTS 0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				57.702						
0.35										
0.34										
0.33										
0.32			48.983							
0.31										
0.3										
0.29										
0.28		51.946								
0.27										
0.26				89.762&92.246						
0.25										
0.24										
0.23										
0.22										
0.21								95.623		
0.2							96.372			
0.19						96.682				
0.18										
0.17							96.749			
0.16			75.459			97.305				
0.15				92.093&91.418					93.116	
0.14										
0.13										
0.12										
0.11							97.888			
0.1			97.975							
0.09					98.299					84.798
0.08			96.138			98.726	97.988		92.982	
0.07										
0.06			97.937							
0.05					97.574&97.753					
0.04						96.137	96.497	88.484&92.674		
0.03			95.596		96.710					
0.02		91.586 90.088	[90.728 & [90.905	90.455	92.518	93.227	91.137	91.691 90.568	89.435 88.425	88.511
0.01										
0.00	84.123									
-.01					78.847					
-.02					73.768	77.998				

TABLE 3. 2. 4

THE EFFICIENCY,ON FORTY-EIGHT DISTRIBUTIONS,OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 1, 2, 2)

CENTRE OF CUTPOINTS

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				26.894						
0.35										
0.34										
0.33										
0.32			26.005							
0.31										
0.3										
0.29										
0.28		32.944								
0.27										
0.26				65.926&71.395						
0.25										
0.24										
0.23										
0.22										
0.21								94.293		
0.2							92.533			
0.19						86.905				
0.18										
0.17							93.771			
0.16			58.352			89.932				
0.15				78.674&77.327					97.129	
0.14										
0.13										
0.12										
0.11							97.099			
0.1			92.336							93.090
0.09					93.393					
0.08			90.534			97.640	98.889		97.934	
0.07										
0.06			95.450							
0.05					99.984&99.990					
0.04						99.698	99.801	95.642&98.268		
0.03			97.839		99.351					
0.02		95.903 94.853	[95.273 & [95.269	95.102	96.981	97.962	96.672	97.160 96.274	96.076 95.112	95.280
0.01										
0.00	90.420									
-.01					85.469					
-.02					80.761	84.648				

TABLE 3. 2. 5

THE EFFICIENCY,ON FORTY-EIGHT DISTRIBUTIONS,OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 1, 1, 1)

P.DIF	CENTRE OF CUTPOINTS 0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				16.047						
0.35										
0.34										
0.33										
			16.792							
0.32										
0.31										
0.3										
0.29										
		23.543								
0.28										
0.27										
				49.685&55.727						
0.26										
0.25										
0.24										
0.23										
0.22										
								87.403		
0.21										
							84.410			
0.2										
0.19						76.033				
0.18										
0.17							87.106			
			47.288							
0.16						80.874				
				67.617&65.993						
0.15									95.802	
0.14										
0.13										
0.12										
0.11							93.024			
0.1			85.741							
										96.977
0.09					87.134					
			84.125			93.592			98.539	
0.08							96 407			
0.07										
			91.012							
0.06										
					98.871&98.734					
0.05										
								98.636&99.802		
0.04						99.591	99.481			
			96.877		98.589					
0.03										
			[96.241			98.946			98.738	
0.02		96.669	&	96.174	97.816			98.813	97.848	98.088
		96.024	[96.071				98.397	98.167		
0.01										
0.00	93.042									
-.01					88.544					
						87.782				
-.02					84.335					

TABLE 3. 2. 6

THE EFFICIENCY,ON FORTY-EIGHT DISTRIBUTIONS,OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 1, 2, 4)

CENTRE OF CUTPOINTS

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				10.518						
0.35										
0.34										
0.33										
			15.670							
0.32										
0.31										
0.3										
0.29										
		27.190								
0.28										
0.27										
				32.998&48.112						
0.26										
0.25										
0.24										
0.23										
0.22										
								51.354		
0.21										
							53.827			
0.2										
0.19						59.357				
0.18										
0.17							62.793			
			48.358							
0.16						64.743				
				61.842&62.409						
0.15									70.982	
0.14										
0.13										
0.12										
0.11							77.084			
0.1			90.407							
										79.126
0.09					85.520					
			87.032			85.046			82.879	
0.08							86.137			
0.07										
			94.177							
0.06										
					95.277&95.769					
0.05										
								92.814&92.734		
0.04						97.453	96.011			
			99.958		99.063					
0.03										
			[98.450			98.548			94.680	
0.02		98.897	&	98.383	99.027			97.670	95.659	95.192
		98.101	[98.350				97.745	97.494		
0.01										
0.00	93.975									
-.01					89.591					
						88.907				
-.02					84.982					

TABLE 3. 2. 7

THE EFFICIENCY, ON FORTY-EIGHT DISTRIBUTIONS, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 1, 2, 2)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				6.498						
0.35										
0.34										
0.33										
0.32			10.088							
0.31										
0.3										
0.29										
0.28		18.734								
0.27										
0.26				22.915&36.298						
0.25										
0.24										
0.23										
0.22										
0.21								39.043		
0.2							41.552			
0.19						47.289				
0.18										
0.17							50.965			
0.16			37.021			52.946				
0.15				49.764&50.465					61.139	
0.14										
0.13										
0.12										
0.11							67.001			
0.1			82.636							
0.09					76.414					74.361
0.08			77.987			76.909	78.501		76.588	
0.07										
0.06			87.504							
0.05					91.680&92.333					
0.04						95.660	92.849	93.890&90.942		
0.03			98.268		97.062					
0.02		99.855	99.816 & 99.876	99.752	99.768	99.053		98.605	95.780 98.009	97.301
0.01		99.748					99.123	99.079		
0.00	97.783									
-.01					94.517					
-.02					90.761	93.907				

TABLE 3. 2. 8

THE EFFICIENCY, ON FORTY-EIGHT DISTRIBUTIONS, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 1, 1, 1)

CENTRE OF CUTPOINTS

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				3.297						
0.35										
0.34										
0.33			5.557							
0.32										
0.31										
0.3										
0.29		11.601								
0.28										
0.27										
0.26					12.862&22.563					
0.25										
0.24										
0.23										
0.22										
0.21								23.333		
0.2							25.504			
0.19						30.943				
0.18										
0.17							33.592			
0.16			25.122			35.925				
0.15					34.422&35.314				43.053	
0.14										
0.13										
0.12										
0.11							49.373			
0.1			70.050							
0.09					61.591					59.608
0.08			64.629			61.159	62.804		61.003	
0.07										
0.06			76.265							
0.05						81.174&82.176				
0.04						87.855	83.156		88.812&81.591	
0.03			92.934		90.078					
0.02		97.952 98.847	[98.535 & [98.561	98.582	96.992	95.116	96.494	95.043 96.756	91.247 96.332	94.840
0.01										
0.00	99.992									
-.01					98.836	98.511				
-.02					96.661					

TABLE 3. 2. 9

THE EFFICIENCY,ON FORTY-EIGHT DISTRIBUTIONS,OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (-1, 2,15,10)

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
				CENTRE OF CUTPOINTS						
0.355				45.120						
0.35										
0.34										
0.33			38.873							
0.32										
0.31										
0.3										
0.29		42.175								
0.28										
0.27				86.936&89.519						
0.26										
0.25										
0.24										
0.23										
0.22								99.099		
0.21							99.871			
0.2										
0.19						98.199				
0.18										
0.17			68.539				99.991			
0.16				89.169&88.018		98.836				
0.15									97.085	
0.14										
0.13										
0.12										
0.11							99.697			
0.1			94.741							87.722
0.09			92.695		96.492	99.629			95.892	
0.08							99.186			
0.07			95.429							
0.06					98.283&98.376					
0.05								90.744&94.602		
0.04			94.594		96.506	96.815	97.050			
0.03						93.859			91.040	
0.02		91.452 90.143	[90.699 & [90.957	90.386	92.666		91.944	92.505 91.393	89.866	90.078
0.01										
0.00	84.900									
-.01					79.771	78.890				
-.02					74.912					

TABLE 3. 2.10

THE EFFICIENCY,ON FORTY-EIGHT DISTRIBUTIONS,OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (-1, 2,10,15)

P.DIF	CENTRE OF CUTPOINTS 0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				57.073						
0.35										
0.34										
0.33			51.384							
0.32										
0.31										
0.3										
0.29		54.706								
0.28										
0.27				92.512&94.213						
0.26										
0.25										
0.24										
0.23										
0.22								96.861		
0.21							98.271			
0.2										
0.19						98.935				
0.18										
0.17			78.100				98.637			
0.16				94.264&93.392		99.308				
0.15									94.194	
0.14										
0.13										
0.12										
0.11							99.175			
0.1			98.034							84.659
0.09			97.185		99.276	99.550			93.461	
0.08							98.704			
0.07			98.440							
0.06					[illegible].449&97.584					
0.05								87.959&92.419		
0.04			95.217		96.359	95.749	96.482			
0.03			[89.899			92.508			88.715	
0.02		90.813	&	89.672	91.771			90.947	87.643	87.710
0.01		89.220	[89.899				90.360	89.761		
0.00	83.032									
-.01					77.602	76.775				
-.02					72.468					

when the midpoint of the cutpoints is about 1.75, falling away on either side. To the left a better estimator can be obtained by giving more weight to the centre and to the right by giving less weight to the centre (see Table 2.6.10).

As expected we find that singly-trimmed estimators achieve their best performance on distributions whose probability difference is smaller again. The singly-trimmed mean reaches its optimum at a probability difference of about 0.04 where its efficiency often reaches 99 percent. If one is concerned about reasonably small departures from normality the singly-trimmed mean is the most robust estimator among those considered. It even performs reasonably well for low-tailed distributions. An estimator which is almost as good on near-normal distributions but which is somewhat better for more severe departures from normality is the one with weight ratios (0,1,2,2).

The mean of course is best at the normal distribution and its quality gradually falls away as the distribution is made more nonnormal. However it can stand probability differences of up to 0.03 reasonably well and is good on the low-tailed distributions. The estimator with weights in the ratios (1,1,2,4) maintains high quality for probability differences up to 0.05 or 0.06 and is also reasonable for low-tailed distributions. The estimator with weight ratios (1,1,2,2) maintains 90 percent efficiency for distributions with probability differences up to about 0.05 and also for the low-tailed distributions.

The best of the estimators with negative coefficients was one with weight ratios (-1,2,15,10). It performs best when the probability difference is about 0·17 and falls away as the probability difference departs from this. Near the normal distribution it is not much worse than the singly-trimmed mean and is better than the singly-trimmed mean for severe departures from normality.

It is interesting that this estimator is more robust than the one with weight ratios (-1,2,10,15), where the weights change monotonicly in each half. Other examples were found where making the weights non-monotonic improved the robustness of an estimator. Perhaps the explanation for this is that weight has been taken from the extreme order statistics in order to cope with the tails of the distribution but that in doing this weight is also removed from the points likely to fall into the useful shoulder region of the distribution, and, to compensate for this, extra weight is given to the third order statistic. It may be worthwhile to explore this inversion property for estimators other than linear ones.

These results are not inconsistent with results others have obtained. For example, Huber in his overall comments on the Princeton robustness study (Andrews et al 1972, p.254) includes the 10 percent, 15 percent and 25 percent trimmed means among the estimates he favours.

Do the results obtained here suggest that the method used to classify the distributions is useful? It seems to me that they do because there is a reasonable continuity in all the tables. Clearly the probability difference explains much of the variation in the quality of an estimator on different distributions. Furthermore the midpoint of the cutpoints seems a useful second variable because it often explains much of the remaining variation in the quality.

3.3 Robustness on Distributions with Variance One

In this section the "robustness" of estimators as measured by their variance on distributions with variance one is considered. I do not consider this measure to be a good one to use to measure robustness directly because, as explained in Section 2.2, it neglects the variation in the variances of the different possible distributions. It is considered here so that the results may be contrasted with the results of 3.2 in order to show up inadequacies in efficiency as a measure.

The estimators considered are the median, the doubly trimmed mean, the singly trimmed mean and the mean. The results are given in Tables 3.3.1 to 3.3.4 and are tabulated in Appendix 5.

Each estimator except the mean has smaller variance on the highly nonnormal distributions (which were seen in Section 2.6 to be highly estimable) than on the near-normal distributions. The trend is naturally more pronounced for the estimators which weight the middle observations more and thus achieve good performance for high-tailed distributions at the expense of good performance for near-normal distributions. The median, for example, achieves a variance of 0.0045 on the most nonnormal distribution but only achieves a variance of 0.1691 on the approximation to the normal distribution.

In the light of these figures on the actual variances of the estimators on various distributions one must ask whether it is worthwhile striving for as high efficiency on the very nonnormal distributions as on the near-normal distributions, since the actual variances of the estimators are so low for the highly nonnormal distributions. The median was found in

TABLE 3. 3. 1

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS OF VARIANCE ONE,
OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 0, 0, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.004						
0.35										
0.34										
0.33										
0.32			0.008							
0.31										
0.3										
0.29										
		0.016								
0.28										
0.27										
				0.017&	0.031					
0.26										
0.25										
0.24										
0.23										
0.22										
								0.035		
0.21										
							0.037			
0.2										
0.19						0.043				
0.18										
0.17							0.049			
			0.033							
0.16						0.050				
				0.045&	0.046					
0.15									0.067	
0.14										
0.13										
0.12										
0.11							0.069			
0.1			0.090							
										0.103
0.09					0.079					
			0.082			0.084			0.094	
0.08							0.088			
0.07										
			0.098							
0.06										
					0.115&	0.116				
0.05										
								0.146&	0.125	
0.04						0.127	0.118			
			0.130		0.126					
0.03										
			0.150			0.143			0.146	
0.02		0.147	&	0.150	0.145			0.146	0.156	0.154
		0.152	0.151				0.149	0.151		
0.01										
0.00	0.169									
-.01					0.181					
						0.182				
-.02					0.191					

TABLE 3. 3. 2

THE VARIANCE, ON FORTY-EIGHT DISTRIBUTIONS OF VARIANCE ONE, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 0, 1, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.007						
0.35										
0.34										
0.33										
0.32			0.014							
0.31										
0.3										
0.29										
0.28		0.028								
0.27										
0.26				0.018& 0.031						
0.25										
0.24										
0.23										
0.22								0.031		
0.21							0.033			
0.2										
0.19						0.040				
0.18										
0.17			0.042				0.043			
0.16				0.047& 0.048		0.046				
0.15									0.058	
0.14										
0.13										
0.12										
0.11							0.063			
0.1			0.089							0.088
0.09			0.084		0.078	0.077			0.082	
0.08							0.080			
0.07			0.097							
0.06					0.104& 0.105					
0.05								0.125& 0.110		
0.04			0.122		0.116	0.114	0.108			
0.03			0.136			0.128			0.128	
0.02		0.134 0.137	& 0.136	0.136	0.131		0.132	0.130 0.134	0.136	0.134
0.01										
0.00	0.149									
-.01					0.157	0.158				
-.02					0.164					

TABLE 3. 3. 3

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS OF VARIANCE ONE,
OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 1, 1, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.026						
0.35										
0.34										
0.33										
0.32			0.041							
0.31										
0.3										
0.29										
0.28		0.062								
0.27										
0.26				0.032&	0.051					
0.25										
0.24										
0.23										
0.22										
0.21								0.033		
0.2							0.038			
0.19						0.051				
0.18										
0.17							0.048			
0.16			0.066			0.056				
0.15				0.064&	0.067				0 056	
0.14										
0.13										
0.12										
0.11							0.066			
0.1			0.102							0.077
0.09					0.088					
0.08			0.096			0.082	0.081		0.077	
0.07										
0.06			0.105		0.103&	0.104				
0.05								0.113&	0.102	
0.04						0.110	0.104			
0.03			0.120		0.114					
0.02		0.127 0.129	0.128 & 0.128	0.128	0.124	0.120	0.123	0.120 0.123	0.116 0.123	0.121
0.01										
0.00	0.134									
-.01					0.140	0.140				
-.02					0.143					

TABLE 3. 3. 4

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS OF VARIANCE ONE,
OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 1, 1, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.125						
0.35										
0.34										
0.33										
0.32			0.125							
0.31										
0.3										
0.29										
0.28		0.125								
0.27										
0.26					0.125& 0.125					
0.25										
0.24										
0.23										
0.22										
0.21								0.125		
0.2							0.125			
0.19						0.125				
0.18										
0.17							0.125			
0.16			0.125			0.125				
0.15					0.125& 0.125				0.125	
0.14										
0.13										
0.12										
0.11							0.125			
0.1			0.125							
0.09					0.125					0.125
0.08			0.125			0.125	0.125		0.125	
0.07										
0.06			0.125							
0.05						0.125& 0.125				
0.04						0.125	0.125		0.125& 0.125	
0.03			0.125		0.125					
0.02		0.125 0.125	0.125 & 0.125	0.125	0.125	0.125	0.125	0.125 0.125	0.125 0.125	0.125
0.01										
0.00	0.125									
-.01					0.125					
-.02					0.125	0.125				

Section 3.2 to be highly efficiency-robust over the whole set of distributions, achieving an efficiency of over 90 percent for highly nonnormal distributions. Would it be better to sacrifice some of this efficiency on highly nonnormal distributions in order to gain efficiency on less estimable distributions? The same question is raised by the figures for the trimmed means both of which achieve lower variance for more nonnormal distributions.

This variation in the estimability of distributions is one of the factors which was claimed in Section 2.2 to have been overlooked in robustness studies. The other factor is also brought out by the figures in this section. The mean has constant variance on distributions of variance one (see Table 3.3.4) and this variance is less than the minimax variance for other estimators (see Tables 3.3.1, 3.3.2 and 3.3.3). Yet the mean is regarded as non-robust. This can only be reasonable if it is believed that the different distributions which could possibly be generating given data have different variances. In the light of Tables 3.3.1 to 3.3.4 it would only be reasonable to use a trimmed mean if it was believed that the highly nonnormal distributions which could be generating the data had larger variance than the near-normal distributions which could be generating the data.

This raises the question of what variance a distribution from a particular family should have in order for data from it to look most like data from, say, the standard normal distribution. Unfortunately I did not have time to attempt a detailed investigation of this using good-of-fit tests. Instead a rough-and-ready method was used to choose the variance of a distribution so that it looked most like the standard normal distribution: each distribution was scaled so that the 95th percentile was the same as the 95th percentile of the standard normal distribution.

The rationale for this is that we should judge the similarity of two distributions on their similarity in some central region and, further, one simple way of saying that two distributions are similar in some central region is to say that they have the same total probability in that region. It seems reasonable to judge the similarity of two distributions on their similarity in some central region because, the probability density being higher in the centre, there is more information there on which to judge the shape of the distribution. This seems to be the viewpoint of robust statistics inasmuch as consideration is given to the *possibility* of high tails and consequent outliers, and estimators are sought which are adequate for all possible tails - as if the density in the tails is not very well known. Standardization of the 95th percentiles makes the distributions (weakly) similar in the middle and allows consideration of different possibilities for the tails.

In Section 3.4 the robustness of linear estimators over such sets of distributions is considered.

3.4 Robustness on Scaled Distributions

In this section the robustness of linear estimators is assessed for distributions which are scaled to have the same 95th percentile as the standard normal distribution. The distributions considered are the same ones considered previously except for the scaling.

Some of the densities have large discontinuities which could make the scaling process unstable, the amount of scaling being quite different depending on whether the 95th percentile fell just before or just after the discontinuity. Thus two distributions which are close together in the classification scheme and thus have similar shapes when their variances are one, may not have similar shapes after being scaled. However if two neighbouring distributions are reasonably continuous they should be scaled similarly so that they are still similar after scaling. Table 3.4.1 shows that neighbouring distributions are similarly scaled except near the top of the table. These very discontinuous distributions are ignored in the following discussion which thus concentrates on the more realistic distributions, those omitted being, in the main, extremely nonnormal. The 17 distributions omitted were distributions (2,1) and all the distributions with probability differences greater than 0.0965 except distribution (8,3). 31 distributions remained. Thus in the discussion a "big" probability difference is a probability difference of about 0·11.

Of the forty-one estimators considered the results are given for twelve of the better ones. The estimators discussed have weights (W[1], W[2],..., W[8]) with W[I] = W[9 - I] and with (W[1], W[2], W[3], W[4]) in the ratios (0,0,0,1), (0,0,1,1), (0,0,2,1), (0,1,1,1), (0,1,1,2), (0,1,2,2), (1,1,1,1), (1,1,1,2), (1,1,2,2), (1,1,2,4), (1,2,2,2) and (-1,2,15,10). The results are given in Tables 3.4.2 to 3.4.13 and are tabulated in Appendix 5. They are now discussed.

TABLE 3.4.1

THE FACTORS BY WHICH FORTY-EIGHT DISTRIBUTIONS MUST BE SCALED TO MAKE THEIR 95TH PERCENTILES 1.645

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				1.640						
0.35										
0.34										
0.33										
0.32			0.962							
0.31										
0.3										
0.29										
0.28		0.867								
0 27										
0.26					1.911& 1.136					
0.25										
0.24										
0.23										
0.22										
0.21								2.432		
0.2							2.340			
0.19						1.739				
0.18										
0.17							1.949			
0.16			0.981			1.441				
0.15					1.249& 1.214				1.805	
0.14										
0.13										
0.12										
0.11							1.408			
0.1			0.912							
0.09					1.112					1.472
0.08			1.001			1.194	1.215		1.415	
0.07										
0.06			1.002							
0.05						1.081& 1.071				
0.04						1.059	1.098		1.183& 1.140	
0.03			0.996		1.019					
0.02		0.996[	0.995 &	1.003	1.007	1.029		1.039	1.082 1.090	1.070
0.01		0.995[	0.970				1.031	1.032		
0.00	1.009									
-.01					0.991					
-.02					0.996	0.991				

TABLE 3. 4. 2

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645,
OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 0, 0, 1)

P.DIF	CENTRE OF CUTPOINTS 0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.012						
0.35										
0.34										
0.33										
0.32			0.007							
0.31										
0.3										
0.29										
0.28		0.012								
0.27										
0.26				0.063&	0.040					
0.25										
0.24										
0.23										
0.22										
0.21								0.206		
0.2							0.204			
0.19						0.130				
0.18										
0.17							0.185			
0.16			0.032			0.103				
0.15				0.070&	0.068				0.218	
0.14										
0.13										
0.12										
0.11							0.137			
0.1			0.075							
0.09					0.098					0.223
0.08			0.082			0.120	0.129		0.188	
0.07										
0.06			0.099							
0.05					0.134&	0.133				
0.04						0.142	0.142	0.204&	0.163	
0.03			0.128		0.131					
0.02		0.146[0.150[	0.149 & 0.142	0.151	0.147	0.151		0.157	0.171 0.185	0.176
0.01							0.158	0.160		
0.00	0.172									
-.01					0.177					
-.02					0.190	0.179				

TABLE 3. 4. 3

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645,
OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 0, 1, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.019						
0.35										
0.34										
0.33										
0.32			0.013							
0.31										
0.3										
0.29										
		0.021								
0.28										
0.27										
				0.065&	0.039					
0.26										
0.25										
0.24										
0.23										
0.22										
								0.180		
0.21										
							0.181			
0.2										
0.19						0.121				
0.18										
0.17							0.165			
			0.040							
0.16						0.096				
				0.073&	0.071					
0.15									0.188	
0.14										
0.13										
0.12										
0.11							0.125			
0.1			0.074							
										0.190
0.09					0.097					
			0.084			0.110			0.164	
0.08							0.118			
0.07										
			0.098							
0.06										
					0.121&	0.120				
0.05										
								0.176&	0.143	
0.04						0.128	0.130			
			0.120		0.121					
0.03										
0.02		0.133 [0.136 [	0.135 & 0.127	0.137	0.133	0.135	0.141	0.140 0.142	0.149 0.162	0.153
0.01										
0.00	0.151									
-.01					0.154					
						0.155				
-.02					0.162					

TABLE 3. 4. 4

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 0, 2, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.024						
0.35										
0.34										
0.33										
0.32			0.017							
0.31										
0.3										
0.29										
0.28		0.027								
0.27										
0.26				0.070&	0.042					
0.25										
0.24										
0.23										
0.22										
0.21								0.178		
0.2							0.180			
0.19						0.123				
0.18										
0.17							0.164			
0.16			0.045			0.097				
0.15				0.077&	0.075				0.185	
0.14										
0.13										
0.12										
0.11							0.125			
-			0 077							
0 09					0.100					0.186
0.08			0 088			0.111	0.119		0.162	
0.07										
0.06			0.101							
0.05					0.121&	0.120				
0.04						0.128	0.130	0.172&	0.141	
0.03			0 122		0.122					
0.02		0.133[	0.134 &	0.137	0.133	0.134		0.139	0.147 0.159	0.151
0.01		0.135[	0 127				0.139	0.141		
0.00	0.149									
-.01					0.151					
-.02					0.159	0.152				

TABLE 3. 4. 5

THE VARIANCE, ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 1, 1, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.069						
0.35										
0.34										
0.33										
0.32			0.038							
0.31										
0.3										
0.29										
0.28		0.046								
0.27										
0.26					0.118& 0.065					
0.25										
0.24										
0.23										
0.22										
0.21								0.197		
0.2							0.207			
0.19						0.154				
0.18										
0.17							0.183			
0.16			0.064			0.115				
0.15					0.099& 0.099				0.183	
0.14										
0.13										
0.12										
0.11							0.132			
0.1			0.085							
0.09					0.109					0.166
0.08			0.096			0.116	0.120		0.155	
0.07										
0.06			0.105							
0.05						0.120& 0.119				
0.04						0.124	0.126	0.158& 0.133		
0.03			0.119		0.119					
0.02		0.126 0.127	0.127 & 0.121	0.129	0.126	0.127		0.130 0.131	0.135 0.146	0.138
0.01							0.130			
0.00	0.137									
-.01					0.137					
-.02					0.142	0.138				

TABLE 3. 4. 6

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 1, 1, 2)

		CENTRE OF CUTPOINTS								
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.046						
0.35										
0.34										
0.33										
0.32			0.026							
0.31										
0.3										
0.29										
0.28		0.033								
0.27										
0.26				0.095& 0.054						
0.25										
0.24										
0.23										
0.22										
0.21								0.189		
0.2							0.195			
0.19						0.139				
0.18										
0.17							0.175			
0.16			0.052			0.107				
0.15				0.087& 0.086					0.184	
0.14										
0.13										
0.12										
0.11							0.128			
0.1			0.079							
0.09					0.102					0.174
0.08			0.089			0.113	0.118		0.157	
0.07										
0.06			0.100							
0.05					0.119& 0.118					
0.04						0.124	0.126	0.163& 0.135		
0.03			0.117		0.118					
0.02		0.126[	[0.128 &	0.130	0.127	0.129		0.132	0.139 0.151	0.143
0.01		0.129[	[0.122				0.133	0.134		
0.00	0.141									
-.01					0.142					
-.02					0.149	0.143				

TABLE 3. 4. 7

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (0, 1, 2, 2)

		CENTRE OF CUTPOINTS								
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.041						
0.35										
0.34										
0.33										
0.32			0.025							
0.31										
0.3										
0.29										
0.28		0.033								
0.27										
0.26					0.089& 0.051					
0.25										
0.24										
0.23										
0.22										
0.21								0.183		
0.2							0.189			
0.19						0.135				
0.18										
0 17							0 170			
0.16			0.052			0.104				
0.15					0.085& 0.084				0.180	
0.14										
0.13										
0.12										
0.11							0.126			
0.1			0.079							
0 09					0.102					0.173
0.08			0.089			0.112	0.117		0.156	
0 07										
0.06			0.100							
0.05						0.118& 0.118				
0.04						0.124	0.126		0.162& 0.135	
0.03			0.118		0.118					
0.02		0.127[0.129[	[0 128 & 0.122	0.130	0.127	0.129		0.132 0.134	0.139 0.150	0.142
0.01							0.133			
0.00	0.141									
-.01					0.142					
-.02					0.148	0.143				

TABLE 3. 4. 8

THE VARIANCE, ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 1, 1, 1)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.336						
0.35										
0.34										
0.33										
			0.116							
0.32										
0.31										
0.3										
0.29										
		0.094								
0.28										
0.27										
				0.457&	0.161					
0.26										
0.25										
0.24										
0.23										
0.22										
								0.739		
0.21										
							0.684			
0.2										
0.19						0.378				
0.18										
0.17							0.475			
			0.120							
0.16						0.260				
				0 195&	0.184					
0.15									0.407	
0.14										
0.13										
0.12										
0.11							0.248			
0.1			0.104							
										0.271
0.09					0.155					
			0 125			0 178			0.250	
0.08							0.185			
0.07										
			0 126							
0 06										
					0.146&	0.143				
0.05										
								0.175&	0.162	
0.04						0.140	0.151			
			0.124		0.130					
0.03										
			0.124			0.132			0.146	
0.02		0.124	&	0.126	0.127			0.135	0.148	0.143
		.124	0.118				0.133	0.133		
0.01										
0.00	0.127									
-.01					0.123					
						0.123				
-.02					0.124					

TABLE 3. 4. 9

THE VARIANCE, ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 1, 1, 2)

	CENTRE OF CUTPOINTS									
P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.225						
0.35										
0.34										
0.33										
0.32			0.079							
0.31										
0.3										
0.29										
0.28		0 067								
0.27										
0.26				0.323&	0.120					
0.25										
0.24										
0.23										
0.22										
0.21								0.547		
0.2							0.513			
0.19						0.291				
0.18										
0.17							0.369			
0.16			0.092			0.204				
0.15				0.153&	0.145				0.330	
0.14										
0.13										
0.12										
0.11							0.205			
0.1			0.092							
0.09					0.133					0.237
0.08			0.109			0.153	0.160		0.218	
0.07										
0.06			0.113							
0.05					0.134&	0.132				
0.04						0.132	0.140	0.170&	0.152	
0.03			0.118		0.123					
0.02		0.122[0.122[	[0.122 & 0.116	0.124	0.124	0.129	0.130	0.131 0.131	0.142 0.147	0.141
0.01										
0.00	0.129									
-.01					0.127					
-.02					0.130	0.127				

TABLE 3. 4. 10

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 1, 2, 2)

CENTRE OF CUTPOINTS

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.171						
0.35										
0.34										
0.33										
0.32			0.064							
0.31										
0.3										
0.29										
0.28		0.058								
0.27										
0.26				0.256& 0.100						
0.25										
0.24										
0.23										
0.22										
0.21								0.442		
0.2							0.420			
0.19						0.247				
0.18										
0.17							0.313			
0.16			0.082			0.176				
0.15				0.135& 0.129					0.287	
0.14										
0.13										
0.12										
0.11							0.183			
0.1			0.088							
0.09					0.125					0.217
0.08			0.104			0.142	0.148		0.199	
0.07										
0.06			0.109							
0.05					0.129& 0.127					
0.04						0.129	0.135	0.166& 0.146		
0.03			0.117		0.121					
0.02		0.122 [0.123 [	0.122 & 0.116	0.124	0.123	0.127		0.130 0.130	0.139 0.146	0.139
0.01							0.129			
0.00	0.130									
-.01					0.128					
-.02					0.132	0.129				

TABLE 3. 4. 11

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 1, 2, 4)

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
				CENTRE OF CUTPOINTS						
0.355				0.105						
0.35										
0.34										
0.33										
0.32			0.041							
0.31										
0.3										
0.29										
0.28		0.040								
0.27										
0.26				0.178&	0.076					
0.25										
0.24										
0.23										
0.22										
0.21								0.336		
0.2							0.324			
0.19						0.197				
0.18										
0.17							0.254			
0.16			0.062			0.144				
0.15				0.108&	0.104				0.247	
0.14										
0.13										
0.12										
0.11							0.159			
0.1			0.081							
0.09					0.111					0.204
0.08			0.093			0.128	0.135		0.184	
0.07										
0.06			0.102							
0.05					0.124&	0.123				
0.04						0.126	0.131	0.167&	0.143	
0.03			0.115		0.118					
0.02		0.123[	0.124 &	0.126	0.124	0.128		0.131	0.141 0.149	0.143
0.01		0.125[	0.118				0.131	0.132		
0.00	0.135									
-.01					0.135					
-.02					0.141	0.136				

TABLE 3. 4. 12

THE VARIANCE, ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645, OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (1, 2, 2, 2)

P.DIF	0.0	CENTRE OF CUTPOINTS 1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.177						
0.35										
0.34										
0.33										
0.32			0.071							
0.31										
0.3										
0.29										
0.28		0.067								
0.27										
0.26				0.257& 0.106						
0.25										
0.24										
0.23										
0.22										
0.21								0.406		
0.2							0.396			
0.19						0.248				
0.18										
0.17							0.301			
0.16			0.089			0.176				
0.15				0.141& 0.136					0.270	
0.14										
0.13										
0.12										
0.11							0 179			
0.1			0.093							
0.09					0.128					0.203
0.08			0.108			0.142	0.146		0.192	
0.07										
0 06			0.113							
0.05					0.130& 0.128					
0.04						0.129	0.135	0.162& 0.143		
0.03			119		0.122					
0.02		0.122[	[0.123 &	0.125	0.124	0.127		0.129	0.137 0.144	0.137
0.01		0.123[	0.117				0.129	0.129		
0.00	0.129									
-.01					0.127					
-.02					0.130	0.127				

TABLE 3. 4. 13

THE VARIANCE,ON FORTY-EIGHT DISTRIBUTIONS SCALED SO THAT THEIR 95TH PERCENTILE IS 1.645,
OF THE LINEAR ESTIMATOR WHOSE FIRST FOUR COEFFICIENTS ARE IN THE RATIO (-1, 2,15,10)

CENTRE OF CUTPOINTS

P.DIF	0.0	1.125	1.25	1.375	1.5	1.625	1.75	1.875	2.0	2.125
0.355				0.025						
0.35										
0.34										
0.33										
0.32			0.017							
0.31										
0.3										
0.29										
0.28		0.026								
0.27										
0.26				0.068&	0.041					
0.25										
0.24										
0.23										
0.22								0.174		
0.21							0.175			
0.2										
0.19						0.119				
0.18										
0.17			0.044				0.160			
0.16						0.094				
0.15				0.075&	0.074				0.181	
0.14										
0.13										
0.12										
0.11							0.123			
0.1			0.077							0.184
0.09			0.087		0.099	0.109			0.159	
0.08							0.117			
0.07			0.100							
0.06					0.121&	0.120				
0.05								0.171&	0.140	
0.04			0.122		0.121	0.127	0.129			
0.03						0.134				
0.02		0.133 0.136	0.135 & 0.127	0.137	0.133		0.139	0.139 0.141	0.147 0.159	0.151
0.01										
0.00	0.150									
-.01					0.152	0.153				
-.02					0.160					

The median is not one of the best estimators, being clearly outperformed by the singly-trimmed mean for example. It does well for big deviations from normality (i.e. for big probability differences) especially if the centre of the cut-points is close to the centre of the distribution. However its quality for near-normal distributions is not nearly as good as that of some other estimators. The trends in the performance of the median are similar to, but more extreme than, the trends in the performance of most of the estimators. To the left of the table the variance steadily decreases with increasing probability difference. The further one goes to the right of the table it seems (from the limited evidence), the more quickly this decrease is reversed - at first the variance decreases with increasing probability difference but then begins to increase as the probability difference is increased. There is a strong tendency for the variance to increase towards the right of the table.

The doubly-trimmed mean is a significant improvement on the median. The estimator with weight ratios (0,0,2,1) is slightly better than the doubly trimmed mean, at least on near-normal distributions. This seems to be an example of the inversion effect referred to earlier. Perhaps double trimming is too severe, and the increase in W[3] compensates for this.

The singly-trimmed mean is perhaps the most robust of the estimators considered. Its quality varies little over the whole set of distributions and varies even less over the columns corresponding to cutpoint centres from 1·5 to 2·0 (where the standard distributions occur). The estimators considered above, which were more heavily trimmed, were better on highly nonnormal distributions than they were on near-normal distributions. We shall see that estimators which give more weight to the extreme observations than does the singly-trimmed mean, tend to be better for near-normal distributions than for highly nonnormal distributions. The singly-trimmed mean is about where the turning point occurs - its variance is approximately the same for the near-normal and the highly nonnormal distributions and its performance is best since it does not sacrifice quality at one place to gain it at another.

The estimators with weight ratios (0,1,2,2) and (0,1,1,2) are both similar to the singly-trimmed mean but are slightly more heavily trimmed. Their performance is similar to that of the singly-trimmed mean but not quite as good, as they lose quality on the near-normal distributions and gain it on the more nonnormal distributions and thus cannot guarantee as small a variance overall.

The group of estimators with weight ratios (1,1,2,4), (1,1,2,2), (1,2,2,2) and (1,1,1,2) give more weight to the extremes. These estimators tend to be better than the singly trimmed mean for near-normal distributions, especially when the centre of the cutpoints is not far out from the centre of the distribution. However the estimators become significantly worse than the singly trimmed mean for more nonnormal distributions - by putting more weight on the extreme observations they gain quality for the near-normal distributions at the expense of quality on the more nonnormal distributions.

The mean is quite a bad estimator being worse than the singly trimmed mean even for distributions which are only slightly nonnormal and becoming significantly worse as the distribution becomes more nonnormal.

The estimator with weight ratios (-1,2,15,10) performs creditably although like other heavily trimmed estimators it loses quality on the near-normal distributions.

It is noticeable that heavy trimming, while of advantage towards the left of the table for very nonnormal distributions, becomes less useful towards the right of the table where singly trimmed estimators are better than doubly trimmed estimators.

If one seeks an estimator which will cope with low tails as well as slightly high tails the trimmed mean is still adequate, at least for the small number of short-tailed distributions considered. However estimators such as (1,1,1,2), (1,1,2,2) and (1,2,2,2) are significantly better. The mean is better still on low tails but loses quality quickly as the tails become more than a little higher than normal. It was found that further small gains in quality could be made for short-tailed distributions by using estimators which give slightly more weight to the extreme observations than to the central observations.

The fact that low-tailed distributions can be handled adequately by estimators that are designed to cope with high-tailed distributions and which do not have a high efficiency for low-tailed distributions, is an indication that if the distribution has lower tails than expected the location parameter is easy to estimate. This is because the two factors referred to earlier, which work against one another for high tails, work together to improve the quality of the estimator for low tails. In other words, as the distribution becomes more low-tailed it becomes less normal and therefore more estimable and, at the same time, it must be scaled by a progressively smaller factor (less than one) in order to make it like the standard normal distribution in the middle (i.e. if the tails are lower than expected the variance of the distribution is correspondingly smaller than expected).

In assessing the worth of the results obtained in this section it must be remembered that the method used to choose the variances of the distributions was primitive. For example the use of the 95th percentile was somewhat arbitrary. When the study was repeated with scaling based on the 70th percentile the singly-trimmed mean was found to be less robust than some more heavily trimmed estimators such as the doubly-trimmed mean. (This is not surprising since one would expect highly peaked distributions to be scaled more if the scaling was based on a lower percentile). Thus the results show only approximately which estimators are most robust. Nevertheless they do show that if one allows for the two factors which efficiency does not allow for (i.e. the variation in the variances and estimabilities of the different possible distributions) the results obtained are not greatly different from those which have been obtained using efficiency as the measure of quality. To do the fine-tuning would require a more sophisticated method of choosing the variances of the distributions, perhaps using goodness-of-fit tests.

3.5 Conclusion

The conclusions about robust estimation drawn from this study are not at variance with the conclusions of other robustness studies. For example the 25 percent trimmed mean, which was found here to be robust, was also considered robust by both Hampel and Huber in their comments on the Princeton study (Andrews et al 1972, p.240 and p.254). The present study strengthens support for these previous results because it has tested them over a different set of distributions, using different measures of quality. It seems that, as suggested in Section 2.2, the two factors omitted in using efficiency as a measure of quality do tend to cancel one another out. Thus efficiency may sometimes be a reasonable approximation to a measure which does allow for these factors.

The main departures of this study from other studies have been in the organization of the distributions and in the measure of quality used. If these ideas are considered useful there is ample scope for their development. Further studies of the dependence of the performance of estimators on the rough shape of the error density function could be made. More distributions could be considered, especially ones with infinite tails. Other sample sizes could be considered. Other types of estimator could be considered - M-estimators for example. The choice of the variance of a distribution of a given shape could be made more sophisticated, perhaps using goodness-of-fit tests. It may be interesting to classify distributions according to the probability mass they have between adjacent pairs of a series of fixed points, without scaling the distributions to have variance one. Distributions could be constructed for which these probabilities differed by varying amounts from the corresponding probabilities for the standard normal distribution. The robustness of estimators could be tested directly on these near-normal distributions without the need to scale the distributions.

CHAPTER 4

FEATURES OF PROBABILITY DENSITY FUNCTIONS WHICH INFLUENCE THE FORM OF GOOD ESTIMATORS

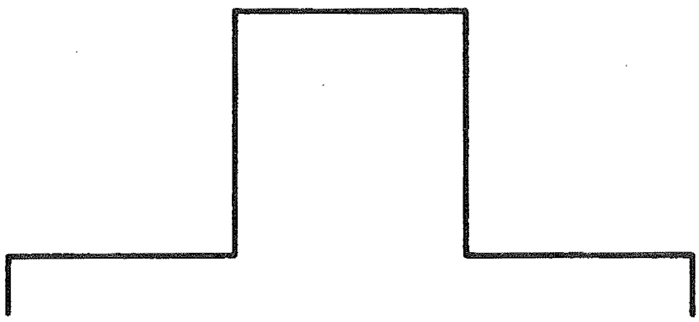
4.1. Introduction

In this chapter an attempt is made to build an intuition about the estimation of the location parameter of a distribution - about how the shape of the density function influences the form of the best linear estimator.

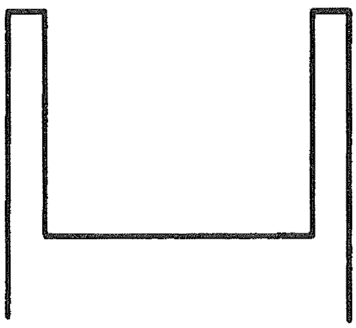
Why does the best estimator for the uniform distribution put all the weight on the extreme points? Given the shape of the normal density function, why does the normal distribution have a best estimator which assigns equal weight to all points and why should this distribution have the special properties that it has? Why is it that good estimators for distributions having higher tails than the normal distribution give little weight to the extreme observations? Why does the best linear estimator for a distribution sometimes have negative coefficients? Which shapes make distributions most estimable?

It is too much to hope that a few features of the shape of the density function will exactly determine the best linear estimator for a distribution. What were sought instead were features of a density's shape which roughly determine the form of the best linear estimator, and which thus allow an understanding of general trends in estimation.

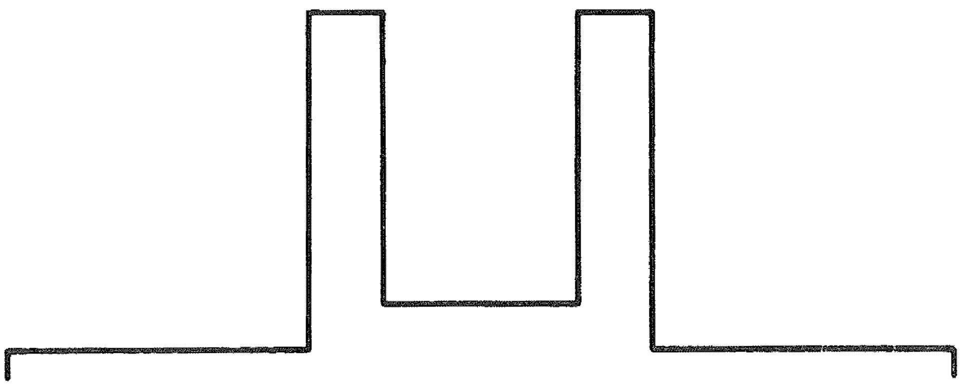
To this end, in Section 4.2, best linear estimators were found for distributions like those in Figure 4.1.



(a) a three-step distribution



(b) a three-step distribution



(c) a five-step distribution

FIGURE 4.1

STEP DISTRIBUTIONS

These will be called step distributions. In Section 4.3 the ideas of Section 4.2 are applied to other distributions including the normal distribution and near-normal distributions.

4.2 The Best Linear Estimators for Step Distributions.

In this section step distributions like those in Figure 4.1 are studied in an attempt to gain an understanding of the effect of the general shape of the probability density function on the estimation of a distribution's location parameter.

The desire for such an understanding was produced by various questions and apparent conflicts. For example the uniform density drops suddenly to zero at the extremes and the best estimator puts all the weight on the extreme points. Is there some relationship between estimation and the shape of a density which explains this? If long low uniform tails are added to the uniform distribution where does the weight in the best estimator go? Why is it that for some distributions the best linear estimator has one or more negative coefficients when this means that if the data point corresponding to one of these coefficients is moved a little in one direction the estimate moves a little in the other direction? If two distributions have the same variance why can estimation sometimes be made more accurate for one than for the other? What features of the shape allow this?

In an endeavour to answer such questions the best linear estimators were calculated for various step distributions. Various features of the distributions were varied systematically in order to see how the form of the best linear estimator depended on these features. Step distributions were used because their primitive nature allows basic features to be varied in a simple way. While the distributions are unrealistic it should be that they exhibit the same patterns as more realistic distributions but in a more extreme, and hence more transparent, way.

While for higher sample sizes estimators may be sensitive to more complex features of a distribution it seems that for small sample sizes it is enough to focus on two factors in order to resolve the apparent conflicts and at least make the basic trends seem reasonable.

Intuitively it seems that, other things being equal, regions where the probability density is high are more useful than regions where the probability density is low. This is simply because in a low-probability-density region a given number of points are likely to be spread over a wider range than the same number of points in a high-probability-density region and will therefore have a greater variance. High probability density in a region should cause order statistics likely to fall into that region to have small variances and therefore, other things being equal, to be weighted more in the best linear estimator. This is the first of the factors which will be considered in an attempt to explain estimation.

Consider however that the mean is the best estimator for the normal distribution. In this case points receive the same weight whether they come from regions of high or low probability density. Also the best estimator for the uniform distribution puts all the weight on the extreme points. These examples show that if there is a tendency to weight points from high-probability-density regions it is far from being the whole story. While preliminary study of estimators for step distributions provided some support for the value of high-density regions it was clear that other factors also influence the form of the best linear estimator. Various things such as the rate of change of the density function were considered as possibilities. Then, in an effort to understand why negative coefficients are sometimes useful, I did some calculations for a simple situation involving covariances; I was seeking an understanding of the general effects of covariances,which might extend to the more complex situation involving order statistics. These calculations suggested a second simple factor influencing the form of estimators which makes the estimators for the normal and uniform distributions seem more reasonable.

Let X and Y be two random variables. Let the mean of each be μ, the variance of X be 1, the variance of Y be σ^2 and the covariance of X and Y be c. Consider linear combinations, $aX + (1-a)Y$, of these two random variables. These have mean μ. It is a simple matter to find that linear combination which has minimum variance and which therefore estimates μ with minimum variance. It is the linear combination for which $a = (\sigma^2 - c)/(1 + \sigma^2 - 2c)$. The variance achieved in this case is $(\sigma^2 - c^2)/(1 + \sigma^2 - 2c)$. Table 4.2.1 gives the values obtained for a and for the minimum variance of $aX + (1 - a)Y$ when $\sigma^2 = 2$ for various values of c.

Let us examine this table, ignoring negative covariances since the covariances of the order statistics which we encounter are not negative. (For conditions under which the covariances of order statistics are non-negative see David 1970, p41, or Bickel 1967). The value of a for the linear combination to have minimum variance gradually increases as the covariance increases until, when the co-variance is greater than 1, a is greater than 1 and therefore the coefficient of Y is negative. The variance of the linear combination increases until the covariance is 1 and then decreases again and is 0 when the correlation is 1.

These results make it clear how negative coefficients can be useful. When X and Y have a high positive correlation the corresponding data points, x and y, will both tend to be on the same side of μ and the only way to form a linear combination with μ as its expected value is to make one of the coefficients negative. In the extreme case when the correlation is 1, μ can be found with probability 1 by a linear combination of x and y since $x - \mu$ will be directly proportional to the distance between x and y. It is not difficult to see why, in this case, if y is moved a little one way the estimate moves a little the other way. Thus we can see that when combining several correlated variables into an estimate, negative coefficients can be used to extract the information residing in the distances between different pairs of points.

There is a second pattern in this table which will

TABLE 4.2.1

The coefficient, a, and the variance of the best estimator of μ of the form $aX + (1-a)Y$ where X and Y are random variables having mean μ, variance 1 and 2 respectively and variance c.

Covariance	a	Variance of $aX + (1-a)Y$
$-\sqrt{2}$	0.586	0
-1.4	0.586	0.007
-1.3	0.589	0.055
-1.2	0.593	0.104
-1.1	0.596	0.152
-1.0	0.600	0.200
-0.9	0.604	0.248
-0.8	0.609	0.296
-0.7	0.614	0.343
-0.6	0.619	0.390
-0.5	0.625	0.438
-0.4	0.632	0.484
-0.3	0.639	0.531
-0.2	0.647	0.576
-0.1	0.656	0.622
0	0.667	0.667
0.1	0.679	0.711
0.2	0.692	0.754
0.3	0.708	0.796
0.4	0.727	0.836
0.5	0.750	0.875
0.6	0.778	0.911
0.7	0.813	0.944
0.8	0.857	0.971
0.9	0.917	0.992
1.0	1.000	1.000
1.1	1.125	0.988
1.2	1.333	0.933
1.3	1.750	0.775
1.4	3.000	0.200
$\sqrt{2}$	3.414	0

be useful in understanding estimation using order statistics. As the covariance increases from 0 to 1 the coefficient, (1 - a), of Y in the best linear combination decreases from 0.333 to 0. Now Y is the random variable with the bigger variance and thus seems the less useful of the two. However we see that within this range of covariances the more independent it is from X the more use it is. If it has a high covariance with X it is not much use (perhaps because then it is not telling us anything that X is not already telling us). If it has a low covariance with X it can be useful despite its high variance (perhaps because it is giving us independent information) and is given weight even at the expense of the more accurate X. Note also that the more this second point is used the more accurate is the best estimator.

Thus we see that covariances can be such that two things can happen in estimation using linear combinations of points. Firstly negative coefficients can be useful. Secondly it can be that less accurate points receive some weight, and that the more independent they are the more they receive.

It seems possible that effects similar to those seen in the simple situation above occur in more complex situations involving more variables. Thus the above calculations suggest a second factor which may influence the form of location estimators using order statistics, and which may help explain the form of the estimators for such as the normal and uniform distributions. This is that order statistics which have low covariances with other order statistics should tend to be more useful. This suggests that order statistics which are more spread out or more distant from the rest of the sample may be more useful. At least for unimodal densities which fall away monotonicly from their maximum, the more extreme order statistics are more spread out and more distant from the rest of the sample (e.g. the first order statistic is more distant from the last order statistic than the middle order statistic is from any order statistic) and so should receive more weight than would otherwise be expected.

This seems to explain why the best estimator for the normal distribution assigns equal weight to all order statistics regardless of whether they come from regions of high or low probability density. The extreme order statistics have greater variance (this can be seen for $n \leqslant 20$ in Teichroew 1962) but compensate for this by being more independent. Presumably the normal density falls away at just the right rate for the effects to cancel one another out so that all order statistics receive equal weight. Lloyd (1952) has shown that for distributions which depend on a location and a scale parameter only, the best linear estimator is the mean if and only if the row sums of the covariance matrix of the order statistics are all equal. Further the normal distribution can be characterized by the property that for each sample size the row sums of the covariance matrix are constant (see Theorem 2.3.4). From these results we can deduce that the best linear estimator is the mean for all sample sizes if and only if the distribution is normal. In other words, for the normal distribution, the increase in the variance of the order statistics near the extremes is compensated for in a precise way by a decrease in the covariances, and this fact can be used to prove that the best linear estimator is the mean. It may be interesting to study the relationship between the row sums of the covariance matrix and the corresponding coefficients of the best linear estimator. It is possible that an index of nonnormality could be obtained from these sums. (One possibility is $\lim_{n\to\infty}\left(\frac{1}{n}\sum_i\left|\sum_j c_{ij} - 1\right|\right)$ where the c_{ij}'s are the covariances.)

To see if the approximate form of linear estimators is indeed explained by the two factors that have been suggested above, the best linear estimators were calculated for various step distributions. It was desired to see how much the form of the estimators was influenced by the following two tendencies:

(i) order statistics likely to fall in regions of high probability density tend to receive more weight because they tend to have smaller variances;

(ii) order statistics which are more isolated, such as those falling towards the extremes of the distribution, tend to receive more weight (because they tend to have smaller covariances with other order statistics).

Distributions like those in (a) and (b) of Figure 4.1 will be called three-step distributions and those like (c) will be called five-step distributions. All distributions considered are symmetric and standardized. Three-step distributions will be described by two parameters: the tail probability and height ratio. The tail probability is the sum of the probability masses in the two extreme regions. The height ratio is (the probability density in the central region/the probability density in the extreme regions). Five-step distributions will be described by four parameters: the middle probability, the middle height ratio, the tail probability and the tail height ratio. The middle probability is the probability mass in the third region. The middle height ratio is (the probability density in the second region/the probability density in the third region). The tail probability is the sum of the probability masses in the two extreme regions. The tail height ratio is (the probability density in the second region/the probability density in the first region).

Results referred to in the discussion are given in tables within the chapter. Details are given in the appendices. The discussion will concentrate on sample sizes, N, of 16 and 32.

Let us now make a preliminary examination of the results to see if they support the value of high probability density regions. First consider the series of three-step distributions with tail probability 0.25 and height ratios from 2 to 100000 (see Tables 4.2.3 to 4.2.7). When the height ratio is 2, so that the density is a little higher in the central region than is the uniform density we find that for sample sizes of 16 and 32 the extremes are weighted less than they are for the uniform distribution. As the height ratio is increased the extremes are weighted less and less and the weight moves more towards the centre until, for the most extreme distribution, the best linear estimator is quite similar

TABLE 4.2.2

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from the uniform distribution, and the extreme point of the distribution

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	0.5	-1.528	0.037	0.5	-1.627	0.0104
2	0	-1.325	0.069	0	-1.522	0.0201
3	0	-1.121	0.097	0	-1.417	0.0292
4	0	-0.917	0.120	0	-1.312	0.0376
5	0	-0.713	0.138	0	-1.207	0.0454
6	0	-0.509	0.152	0	-1.102	0.0525
7	0	-0.306	0.161	0	-0.997	0.0590
8	0	-0.102	0.166	0	-0.892	0.0648
9				0	-0.787	0.0700
10				0	-0.682	0.0745
11				0	-0.577	0.0784
12				0	-0.472	0.0817
13				0	-0.367	0.0843
14				0	-0.262	0.0862
15				0	-0.157	0.0875
16				0	-0.052	0.0882

Variances of the estimators

N=16	N=32
0.0196	0.0053

The left endpoint of the distribution

-1.732

TABLE 4.2.3

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a three-step distribution with tail probability 0.25 and height ratio 2, and the endpoints of the regions of the distribution.

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	0.352	-1.632	0.090	0.411	-1.795	0.0329
2	-0.010	-1.324	0.118	-0.026	-1.609	0.0566
3	0.040	-1.070	0.117	-0.025	-1.436	0.0672
4	0.060	-0.854	0.116	-0.010	-1.281	0.0668
5	0.038	-0.658	0.121	0.012	-1.147	0.0616
6	0.015	-0.468	0.129	0.041	-1.029	0.0572
7	0.004	-0.281	0.136	0.042	-0.922	0.0558
8	0.001	-0.094	0.140	0.029	-0.821	0.0571
9				0.017	-0.723	0.0599
10				0.006	-0.626	0.0630
11				0.001	-0.530	0.0661
12				0.000	-0.434	0.0688
13				0.001	-0.337	0.0710
14				-0.000	-0.241	0.0726
15				-0.001	-0.145	0.0737
16				0.001	-0.048	0.0743

Variances of the estimators

N=16	N=32
0.042	0.015

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2
-1.987	-1.192

TABLE 4.2.4

The coefficients, W[I], and variance of the best linear estimator, and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a three-step distribution with tail probability 0.25 and height ratio 10, and the endpoints of the regions of the distribution.

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	0.016	-1.936	0.547	0.037	-2.351	0.2462
2	-0.039	-1.225	0.493	-0.014	-1.847	0.3884
3	-0.040	-0.784	0.261	-0.020	-1.407	0.3909
4	0.010	-0.535	0.114	-0.025	-1.061	0.2952
5	0.170	-0.382	0.059	-0.028	-0.815	0.1802
6	0.238	-0.264	0.045	-0.021	-0.652	0.0958
7	0.112	-0.157	0.043	0.014	-0.545	0.0496
8	0.032	-0.052	0.044	0.104	-0.469	0.0294
9				0.189	-0.407	0.0224
10				0.157	-0.350	0.0207
11				0.073	-0.296	0.0209
12				0.025	-0.242	0.0214
13				0.008	-0.188	0.0221
14				0.001	-0.134	0.0226
15				-0.002	-0.081	0.0229
16				0.001	-0.027	0.0231

Variances of the estimators

N=16	N=32
0.031	0.012

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2
-2.883	-0.665

TABLE 4.2.5

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a three-step distribution with tail probability 0.25 and height ratio 100, and the endpoints of the regions of the distribution

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	-0.002	-2.011	1.1586	-0.000	-2.616	0.543923
2	-0.004	-0.999	0.9633	-0.001	-1.869	0.843147
3	-0.005	-0.430	0.4350	-0.002	-1.227	0.820724
4	-0.006	-0.177	0.1339	-0.002	-0.738	0.585457
5	-0.007	-0.081	0.0317	-0.003	-0.415	0.324631
6	0.003	-0.044	0.0066	-0.003	-0.228	0.145927
7	0.107	-0.024	0.0018	-0.004	-0.133	0.055181
8	0.413	-0.008	0.0011	-0.004	-0.088	0.018197
9				-0.002	-0.066	0.005507
10				0.013	-0.054	0.001735
11				0.083	-0.045	0.000755
12				0.194	-0.036	0.000538
13				0.156	-0.028	0.000504
14				0.057	-0.020	0.000505
15				0.013	-0.012	0.000511
16				0.004	-0.004	0.000515

Variance of the estimator

N=16	N=32
0.000808	0.000297

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2
-3.409	-0.099

TABLE 4.2.6

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a three-step distribution with tail probability 0.25 and height ratio 1000, and the endpoints of the regions of the distribution

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	-0.0002	-2.004	1.2517	-0.00005	-2.6331	0.5899840
2	-0.0004	-0.955	1.0329	-0.00011	-1.8553	0.9130111
3	-0.0005	-0.371	0.4601	-0.00017	-1.1872	0.8859002
4	-0.0007	-0.119	0.1383	-0.00023	-0.6812	0.6286047
5	-0.0012	-0.034	0.0312	-0.00027	-0.3489	0.3456840
6	-0.0044	-0.010	0.0056	-0.00031	-0.1603	0.1534561
7	-0.0152	-0.003	0.0008	-0.00035	-0.0677	0.0569142
8	0.5225	-0.001	0.0001	-0.00040	-0.0279	0.0181380
9				-0.00045	-0.0127	0.0050707
10				-0.00047	-0.0072	0.00012616
11				-0.00027	-0.0050	0.0002838
12				0.00207	-0.0039	0.0000607
13				0.02329	-0.0029	0.0000153
14				0.13172	-0.0020	0.0000071
15				0.22373	-0.0013	0.0000058
16				0.12227	-0.0004	0.0000056

Variances of the estimators

N=16	N=32
0.0000747	0.0000039

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2
-3.459	-0.010

TABLE 4.2.7

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a three-step distribution with tail probability 0.25 and height ratio 100000, and the endpoints of the regions of the distribution

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	0.00002	-2.0024	1.2622	-0.0000005	-2.634548	0.59521865
2	-0.00002	-0.9496	1.0408	-0.0000012	-1.853332	0.92094218
3	-0.00002	-0.3643	0.4630	-0.0000017	-1.182486	0.89328382
4	-0.00005	-0.1129	0.1388	-0.0000022	-0.674564	0.63347748
5	-0.00040	-0.0283	0.0312	-0.0000027	-0.341206	0.34805391
6	-0.00343	-0.0058	0.0056	-0.0000031	-0.152485	0.15430773
7	-0.01577	-0.0010	0.0008	-0.0000035	-0.060190	0.05712644
8	0.51965	-0.0001	0.0001	-0.0000040	-0.021027	0.01816067
9				-0.0000046	-0.006533	0.00505921
10				-0.0000056	-0.001827	0.00125088
11				-0.0000199	-0.000478	0.00027639
12				-0.0000446	-0.000132	0.00005476
13				-0.0003893	-0.000048	0.00000974
14				-0.0037255	-0.000024	0.00000156
15				-0.0147702	-0.000013	0.00000022
16				0.5189686	-0.000004	0.00000003

Variances of the estimators

N=16	N=32
0.00006583	0.00000002

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2
-3.46405	-0.00010

to the median. (I will comment later on the decrease in the central coefficient for N = 16 in moving from the second-to-last to the last distribution). Thus in this series of distributions we see that the higher the probability in the central region and the narrower the region, the more the weight in the best estimator is concentrated in the centre. Except for the first two distributions,where the extreme order statistics receive positive weight, the order statistics receiving positive weight all have expected values lying in the high density region of the distribution.

The five-step distributions (Tables 4.2.9 to 4.2.12) all have tail probability 0.05, tail height ratio 1000 and middle height ratio 100. The middle probability varies from 0.05 to 0.45 so that the regions of high probability are moved progressively further out from the centre. Table 4.2.8 gives the results for a three-step distribution with the same tails. In this series the weight is given to those order statistics which are expected to fall in the regions of high probability density. Consider the figures for sample size 16. For each of the first two distributions the only two order statistics receiving positive weight are the two whose expected values are in the high-probability-density region. In the third distribution positive weights are given to the two order statistics expected to lie in the high-probability region and to one (which receives only a small positive weight) expected to lie just outside this region. In the fourth distribution no order statistic has its expected value in the

TABLE 4.2.8

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a three-step distribution with tail probability 0.05 and height ratio 1000, and the endpoints of the regions of the distribution

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	-0.0017	-1.460	5.11924	-0.000804643	-2.492	6.982283
2	-0.0023	-0.272	0.59767	-0.001085054	-0.671	1.898334
3	-0.0012	-0.111	0.04371	-0.001272290	-0.218	0.301003
4	0.0497	-0.080	0.00326	-0.001097168	-0.127	0.036403
5	0.3264	-0.062	0.00114	0.005519321	-0.106	0.003923
6	0.1212	-0.044	0.00115	0.099797355	-0.096	0.000697
7	0.0076	-0.026	0.00121	0.290872508	-0.086	0.000465
8	0.0003	-0.009	0.00125	0.096946825	-0.077	0.000488
9				0.010299684	-0.068	0.000526
10				0.000773342	-0.059	0.000560
11				0.000047526	-0.050	0.000589
12				0.000002480	-0.041	0.000614
13				0.000000110	-0.032	0.000633
14				0.000000005	-0.023	0.000648
15				-0.000000001	-0.014	0.000657
16				0.000000000	-0.005	0.000662

Variances of the estimators

N=16	N=32
0.00072	0.00026

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2
-7.649	-0.143

TABLE 4.2.9

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a five-step distribution with tail probability 0.05, tail height ratio 1000, middle probability 0.05 and middle height ratio 100, and the endpoints of the regions of the distribution

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	-0.003	-1.588	2.661	-0.0005	-2.332	3.6293
2	-0.005	-0.732	0.311	-0.0007	-1.019	0.9867
3	0.156	-0.613	0.025	-0.0010	-0.693	0.1565
4	0.374	-0.582	0.014	-0.0017	-0.627	0.0189
5	-0.003	-0.538	0.042	0.0097	-0.612	0.0021
6	-0.008	-0.453	0.106	0.3096	-0.605	0.0004
7	-0.005	-0.310	0.196	0.2129	-0.598	0.0005
8	-0.005	-0.111	0.268	-0.0061	-0.590	0.0014
9				-0.0048	-0.581	0.0040
10				-0.0032	-0.567	0.0108
11				-0.0028	-0.543	0.0259
12				-0.0026	-0.504	0.0548
13				-0.0024	-0.440	0.1006
14				-0.0022	-0.347	0.1597
15				-0.0021	-0.223	0.2179
16				-0.0020	-0.077	0.2549

Variances of the estimators

N=16	N=32
0.00584	0.00016

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2	R=3
-6.050	-0.639	-0.541

TABLE 4.2.10

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a five-step distribution with tail probability 0.05, tail height ratio 1000, middle probability 0.1 and middle height ratio 100, and the endpoints of the regions of the distributions

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	-0.0031	-1.503	1.333	-0.00053	-2.030	1.8178
2	-0.0004	-0.897	0.156	-0.00077	-1.101	0.4942
3	0.3170	-0.811	0.016	-0.00116	-0.869	0.0784
4	0.2191	-0.781	0.021	-0.00249	-0.823	0.0095
5	-0.0195	-0.726	0.072	0.04361	-0.812	0.0010
6	-0.0069	-0.612	0.181	0.42197	-0.807	0.0003
7	-0.0033	-0.418	0.335	0.06805	-0.802	0.0007
8	-0.0028	-0.150	0.456	-0.01464	-0.795	0.0022
9				-0.00483	-0.785	0.0069
10				-0.00215	-0.768	0.0188
11				-0.00153	-0.737	0.0447
12				-0.00130	-0.681	0.0930
13				-0.00117	-0.593	0.1681
14				-0.00107	-0.465	0.2631
15				-0.00101	-0.298	0.3554
16				-0.00098	-0.103	0.4133

Variances of the estimators

N=16	N=32
0.00629	0.00013

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2	R=3
-4.661	-0.831	-0.766

TABLE 4.2.11

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a five-step distribution with tail probability 0.05, tail height ratio 1000, middle probability 0.25 and middle height ratio 100, and the endpoints of the regions of the distributions

I	W[I] for N=16	EXP $[X_i]$ for N=16	VAR $[X_i]$ for N=16	W[I] for N=32	EXP $[X_i]$ for N=32	VAR $[X_i]$ for N=32
1	-0.004	-1.362	0.355	-0.00056	-1.633	0.4834
2	0.059	-1.047	0.043	-0.00103	-1.154	0.1314
3	0.446	-0.995	0.011	-0.00257	-1.035	0.0208
4	0.030	-0.955	0.035	0.01193	-1.011	0.0025
5	-0.018	-0.871	0.103	0.34806	-1.005	0.0004
6	-0.007	-0.713	0.225	0.18248	-1.001	0.0007
7	-0.003	-0.473	0.374	-0.02171	-0.996	0.0024
8	-0.002	-0.166	0.481	-0.00904	0.985	0.0072
9				-0.00306	-0.964	0.0183
10				-0.00142	-0.926	0.0405
11				-0.00089	-0.863	0.0780
12				-0.00065	-0.770	0.1322
13				-0.00051	-0.644	0.1983
14				-0.00041	-0.487	0.2657
15				-0.00033	-0.303	0.3209
16				-0.00029	-0.103	0.3519

Variances of the estimators

N=16	N=32
0.00544	0.00018

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2	R=3
-2.990	-1.015	-0.988

TABLE 4.2.12

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a five-step distribution with tail probability 0.05, tail height ratio 1000, middle probability 0.45 and middle height ratio 100, and the endpoints of the regions of the distribution

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	0.0024	-1.361	0.144	-0.00097	-1.53526	0.1953
2	0.2628	-1.154	0.022	-0.00305	-1.23060	0.0531
3	0.2916	-1.093	0.024	0.01240	-1.15449	0.0085
4	-0.0200	-1.009	0.067	0.30467	-1.13791	0.0016
5	-0.0162	-0.866	0.138	0.23081	-1.12984	0.0022
6	-0.0085	-0.663	0.220	-0.01959	-1.11661	0.0063
7	-0.0063	-0.415	0.286	-0.01320	-1.09135	0.0155
8	-0.0057	-0.141	0.321	-0.00462	-1.04822	0.0317
9				-0.00180	-0.98303	0.0554
10				-0.00094	-0.89460	0.0842
11				-0.00072	-0.78501	0.1138
12				-0.00068	-0.65864	0.1397
13				-0.00067	-0.52061	0.1593
14				-0.00062	-0.37543	0.1724
15				-0.00054	-0.22642	0.1799
16				-0.00047	-0.07564	0.1832

Variances of the estimators

N=16	N=32
0.00823	0.00066

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2	R=3
-2.39766	-1.14234	-1.12979

high-density region but the only two order statistics receiving positive weight are the two expected to lie on either side of this region. Moving through the series we find that as the high-probability region is moved out the weight in the best linear estimator moves out too. It is also worth noting that the order statistics which receive the most weight are generally those with the smallest variances.

On the whole, then, the figures for these two series provide support for the belief that order statistics likely to fall into regions of high probability density tend to receive more weight.

Let us now examine the results in more detail to see if they are explained by the two factors suggested above. Consider the three-step distributions with tail probability 0.25, and their best estimators for sample size 32. When the height ratio is 2 most of the weight is still on the extreme observations but the other observations receiving significant weight are those just inside the jump in the density. Notice that of the order statistics with variance around 0.6 the ones furthest out tend to receive more weight. For example W[6] is greater than W[8] even though the variance of X_6 is greater than the variance of X_8. For X_6 the sum of the covariances is somewhat less than it is for X_8. When the height ratio is 10 almost all the weight has moved in from the extremes. Moving out from the centre we see that the variance of the order statistics at first decreases and then begins to increase. Most of the weight goes to the last order statistics before this increase becomes rapid. Note that, for example, the sum of the covariances is less for X_9 than it is for X_{13}. There is a similar pattern when the height ratio is 100. Of the five central order statistics, which all have similar variances, it is the two extreme ones which get most of the weight and it is only when the variances of the order statistics begin to increase more rapidly that the weight stops increasing as one goes out. When the height ratio is 1000 we see that the variance of the order statistics increases slightly as we move from a central one, X_{16}, to the next one out. However this increase is not enough to offset the effect of X_{15}'s being further out, so that W[15] is greater than W[16]. The increase in the variance is greater in moving to X_{14}, so that the weight decreases but it is still greater than it is for the central order statistic. By the time the height ratio is 100000, X_{16} has a much smaller variance than any other order statistic and is the only one to receive positive weight.

There is a similar pattern for sample size 16 except that here the weight reaches the centre sooner. When the height ratio is 10 the three central order statistics (and of course the corresponding ones in the other half of the sample) have similar variances but it is the most extreme of them which receives the most weight, and the next order statistic out is the next most heavily weighted even though its variance is somewhat larger.

The best linear estimator for a distribution with tail probability 0.05 and height ratio 1000 is shown in Table 4.2.8. For sample size 16 most of the weight goes to the furthest out of the order statistics with small variance. For sample size 32 most of the weight goes to the order statistic, X_7, having smallest variance. Notice that X_6 is the second most heavily weighted order statistic despite the fact that its variance is bigger than that of all the order statistics inside it.

How is the spread of the weight related to the form of the distribution itself? Except for the extreme order statistic in the first two distributions, the order statistics receiving positive weight have expected values lying inside the high probability density region of the distribution. For the first distribution the order statistic receiving maximum weight (apart from the extreme) is the third one in from the jump in the density function. In the next three distributions the maximum is successively the fourth, fifth and sixth one in from the jump, and in the last distribution it is the very middle one. Thus it seems that the bigger the jump is, the more the weight retreats from it.

The pattern is clear: the weight tends to go to the most extreme of the order statistics which have reasonably small variance. Hence the results lend support to the theory that the two factors mentioned above roughly determine the weight distribution in the best linear estimator.

The theory could be tested further by considering more distributions. For example five-step distributions in which the high density region is moved around rather more could be considered. It should be noted that a small deviation from the above patterns was found for two distributions shaped like (b) in Figure 4.1. For sample size 32 the weight on the extreme order statistic was slightly less than 0.5, with the order statistics just inside the extreme one receiving small positive weights.

In the series of three-step distributions with tail probability 0.25 we saw that as the density in the centre is made higher relative to the density at the extremes the weight tended to be put more on the central observations so that the best estimator eventually became quite like the median. However notice that, for sample size 16, W[8] is 0.5225 when the height ratio is 1000 but decreases to 0.51965 when the height ratio is increased to 100000. Perhaps what is happening is that as the central density is made very high the variance of the extreme order statistics becomes so much greater than that of the central order statistics that the extreme order statistics cannot even be useful through their correlation with the central order statistics and they lose even the negative weight they had.

There is another obvious influence on the form of the best linear estimators for the distributions we have considered above: the extreme order statistic at each end tends to receive extra weight because of the density's being nonzero on only a finite domain. Why is it that this occurs?

It may be because a distribution whose density drops to zero at its extremes can have a high density at its extremes so that the two factors we have been considering support one another more. In distributions which tail away to infinity, the advantage an extreme point has of being far away from the crowd is nullified by the low probability in the tails. However if a distribution is nonzero on only a finite domain the probability can still be high at the extremes so that the extreme order statistics have the advantage of low covariances without the disadvantage of high variances. Indeed a density's being nonzero on only a finite domain seems to cause extreme order statistics to have lower variance than central ones. For example consider the variances of the order statistics from the uniform distribution shown in Table 4.2.2. These variances increase steadily towards the centre. (It is easy to prove that this occurs for the uniform distribution for all sample sizes.) There are other distributions for which the variance of the extreme order statistic is small. For example consider the distributions in Table 4.2.3 to 4.2.7: in some cases we find that the variance of the order statistics increases as we approach the extreme but then the variance decreases in going from the second most extreme order statistic to the most extreme one, and in other cases, as we move to the extreme order statistic, we find that the increase in the variance is less then expected or the decrease in the variance is more than expected. It is also possible that the covariance effect is significantly greater for the order statistics at each extreme because each of these has an immediate neighbour on only one side and is therefore further away on the average from the other points,

It was noticed that for some of these densities which are nonzero on only a finite domain, the tendency to put weight on the extreme observations became stronger with increasing sample size. This is illustrated in Table 4.2.13 where the best linear estimator is given for samples of sizes 2,4,6,8,10,16 and 32 from a three-step distribution. These figures prompt the speculation that for any distribution which is nonzero on only a finite domain the asymptotically best linear estimator is the midrange. While figures in the earlier tables of this section show that this effect is often very small for small sample sizes, it is possible that it becomes dominant as the sample size increases.

4.3 The Effect of Features of the Density Function on Estimation for Other Distributions.

In this section an attempt is made to use the understanding of estimation developed in section 4.2 to understand estimation for other distributions, in particular for the normal distribution and near-normal distributions.

The normal distribution is special in several ways. For one thing it is the distribution which has the mean, a very natural estimator, as its best estimator. Also under a wide variety of conditions the distribution of sums of random variables tends to the normal distribution by the central limit theorem. Furthermore estimators in many situations are asymptotically normally distributed.

It is natural to ask if there is one special thing about the normal distribution which explains all the other special things. Recall that in Section 2.3 we saw that

TABLE 4.2.13

The coefficients, W[I], of the best linear estimator for sample sizes, N, of 2,4,6,8,10,16 and 32 from a three-step distribution with tail probability 0.5 and height ratio 2.

I	W[I] for N=2	W[I] for N=4	W[I] for N=6	W[I] for N=8	W[I] for N=10	W[I] for N=16	W[I] for N=32
1	0.5	0.3659	0.3462	0.3575	0.3742	0.4152	0.4501
2		0.1341	0.0190	-0.0250	-0.0360	-0.0256	-0.0022
3			0.1348	0.0443	-0.0030	-0.0292	-0.0051
4				0.1232	0.0593	-0.0199	-0.0090
5					0.1055	0.0076	-0.0131
6						0.0420	-0.0160
7						0.0572	-0.0162
8						0.0528	-0.0121
9							-0.0020
10							0.0127
11							0.0262
12							0.0307
13							0.0255
14							0.0164
15							0.0089
16							0.0051

the normal distribution is the hardest distribution of all to estimate. So perhaps the central limit theorem can be interpreted as saying that if data from a lot of different distributions, each with its own structure which may be helpful for estimation, are mixed together the structure and the information that goes with it are lost in the mix and the errors degenerate to the lowest form: the normal distribution. Thus the special thing about the normal distribution is that it is a groundstate - it is the most structureless and least informative distribution.

The fact that the normal distribution is degenerate and uninformative seems to explain why the mean is its best estimator. The normal distribution has no structure which helps in estimation - there are no special points in the distribution which give extra information. Thus its best estimator is structureless and does not emphasize any points above others. Compare, for example, the uniform distribution where information can be extracted from the extremes. Perhaps the mean can be considered the most amorphous estimator, using only background information about the general position of the points rather than using special features of the data to wring out extra information. Consequently its quality is constant. Other estimators which are sensitive to special features in the data gain extra quality when these features are present but lose when they are not.

It has seemed a puzzle to me why the normal distribution which has so many special properties should have a density which looks in no way special or extreme. However in the light of section 4.2 and of the observation that the normal distribution is the most structureless and uninformative distributuon it no longer seems puzzling. It appears that the normal density falls away at just the right rate to balance out the two factors considered in section 4.2. The probability density is not so high in the centre that points there have a variance low enough that they provide more information than the more independent points further out. Similarly the density away from the centre is not high enough that the more independent points out there provide more information than those in the centre. Thus it does not seem strange that a distribution whose density has the shape that the normal distribution has is the least estimable distribution.

If the normal distribution is the least estimable distribution in what ways would we expect changes to it to improve estimability? One possible way is to move some probability mass from the shoulder of the distribution into the middle, at the same time moving a little probability mass from the shoulder out into the tails to keep the variance of the distribution constant. An estimator which weights the middle observations more is then used to take advantage of this high probability density in the middle. This seems to be what is happening when trimmed estimators are used on distributions with higher than normal tails.

Sarhan (1955) studies the best linear estimators for distributions of various shapes and he points out a sequence in the variation of the best linear estimator.

He writes: "It seems that the full sequence is missing its natural extension and the complete sequence should read:

(a) negative weights in the middle and large positive weights at the tails,

(b) zero weights in the middle and equal weights at the tails,

(c) less weights in the middle than at the tails,

(d) equal weights throughout,

(e) more weight in the centre and less weights in tails, but all positive weights,

(f) middle observations receive all the weight, others nothing,

(g) middle observations receive more than unity and tails take on negative weights.

This is the sequence which might be anticipated. The results show that (a) is U-shaped; (b) is rectangular; (c) is triangular or parabolic; (d) is normal; (e) is double exponential; (f) is the case where the median gets all the weight, which is like a double exponential but not exactly. For (g) the author does not know any example at this time...".

We have seen distributions which fit into most places in this series, including the place which Sarhan was unable to fill. Table 4.3.1 shows the best estimator for a distribution, shaped like the one in Figure 4.1(b), which fits into (a). Table 4.2.3 shows a distribution which fits into (c). The three low-tailed distributions in Table 2.6.10 also fit into (c). Table 2.6.10 also shows many distributions which fit into (e) and (g).

TABLE 4.3.1

The coefficients, W[I], and variance of the best linear estimator and the expected values and variances of the order statistics, X_i, for samples of sizes 16 and 32 from a three-step distribution with tail probability 0.5 and height ratio 0.01, and the endpoints of the regions of the distribution

I	W[I] for N=16	EXP[X_i] for N=16	VAR[X_i] for N=16	W[I] for N=32	EXP[X_i] for N=32	VAR[X_i] for N=32
1	0.53553	-1.227	0.001	0.520115	-1.231	0.000005
2	-0.02894	-1.210	0.007	-0.015232	-1.229	0.000041
3	-0.00443	-1.161	0.032	-0.002590	-1.227	0.000265
4	-0.00112	-1.058	0.085	-0.000866	-1.223	0.001209
5	-0.00042	-0.892	0.160	-0.000464	-1.212	0.004146
6	-0.00024	-0.671	0.235	-0.000283	-1.192	0.011218
7	-0.00019	-0.415	0.289	-0.000206	-1.154	0.024748
8	-0.00018	-0.140	0.315	-0.000147	-1.095	0.045679
9				-0.000112	-1.012	0.072260
10				-0.000083	-0.908	0.100414
11				-0.000057	-0.787	0.125731
12				-0.000037	-0.653	0.145458
13				-0.000021	-0.513	0.159073
14				-0.000011	-0.368	0.167552
15				-0.000005	-0.221	0.172287
16				-0.000002	-0.074	0.174374

Variances of the estimators

N=16	N=32
0.000441	0.000002

The left endpoint, P[R], of the Rth region of the distribution

R=1	R=2
-1.232	-1.220

The patterns in Sarhan's series can be explained in terms of the ideas of section 4.2. The weight tends to go where the probability density is high but also has a tendency to go to the extremes. For the U-shaped distribution at the beginning of the series, when these two effects act in concert, the weight goes to the extremes. Moving through the series weight moves towards the centre as probability mass is moved towards the centre until at the normal distribution, where the effects balance, the weight is evenly spread. From then on, as even more weight is put into the centre, the high density effect becomes more dominant and more and more weight is placed on the central observations.

We can go beyond Sarhan's series and find distributions whose best estimators assign their weight in other ways. Indeed it seems that we can cause most of the weight to be put wherever we like, simply by making the probability density high enough at the appropriate place. This is illustrated by the estimators for the five-step distributions whose best linear estimators are displayed in Tables 4.2.9 to 4.2.12.

It should be remembered that some of these patterns depend, at least in degree, on sample size.

The patterns in Table 2.6.10 can also be understood in terms of the ideas of section 4.2. In this table there is a tendency for weight to go to the centre as the probability difference is increased. The explanation for this appears to be that as the probability difference is increased the probability density in the middle becomes higher and the best estimator weights the central observations more in order to take advantage of this. There is also a tendency in the table for weight to go to the centre as the midpoint of the cutpoints is moved closer to the centre of the distribution. The explanation for this appears to be that, as the midpoint of the cutpoints is moved closer to the centre of the distribution, the region of high probability density in the centre becomes narrower and higher, and so an estimator must push the weight more towards the centre in order to take advantage of the high density.

How does the estimability of distributions depend on their shape? Table 4.3.2 shows the variances of (standardized) distributions which Sarhan fitted into his series. (The figures are taken from Sarhan (1955)). In this table of low-tailed distributions the more nonnormal a distribution is the more estimable it is. Further evidence of this is provided by Table 4.3.3 which displays the estimabilities of other low-tailed distributions which fit into (a), (b), (c) and (d) of Sarhan's series. As was noted in section 2.6 it is also true for high-tailed distributions that the more nonnormal they are the more estimable they are. Ample evidence for this is provided in Table 2.6.11. It seems that for nonnormal distributions one or both of the effects introduced in section 4.2 becomes strong and can be used to achieve accurate estimation.

4.4 Conclusion

The best linear location estimators for various

TABLE 4.3.2

Variances of the best linear estimators for samples of size 5 from various standardized distributions

Distribution	Variance of estimator
U-shaped	.0675462
Uniform	.1428371
Parabolic	.1790064
Triangular	.1934059
Normal	.2000000

TABLE 4.3.3

Variances of the best linear estimators for samples of size 8 from various standardized distributions

Distribution	Variance of estimator
Three-step distribution with tail probability 0.05 and height ratio 0.01	0.0142
Uniform	0.0667
Three-step distribution with tail probability 0.25 and height ratio 2	0.1003
Distribution (7,9) from Chapter 2	0.1208
Distribution (4,8) from Chapter 2	0.1231
Distribution (7,8) from Chapter 2	0.1235
Normal	0.1250

Note that the distributions from Chapter 2 are the three low-tailed distributions used there.

simple distributions have been exhibited in this chapter and it is hoped that the patterns revealed aid in the understanding of estimation for other distributions.

The patterns may vary with sample size. For example the effect of a probability density function's being nonzero on only a finite domain increases with sample size. (It was partly to reduce the distortion due to this effect that a sample size of 8 was used in Chapters 2 and 3.) Discontinuities in the density function can also become important asymptotically. It seems to be possible to prove various asymptotic results in this area and I hope to do so in the near future.

It is likely that the broad patterns hold for estimators other than linear combinations of order statistics although in some situations other sorts of estimators should be able to extract information denied to linear combinations of order statistics. For example the best linear estimators for three-step distributions with a high probability density in the central region give most weight to some of the more extreme order statistics expected to fall in the high density region. A better estimate would probably be obtained if weight was put on those points in a particular sample which seemed to be just inside the high density region than if weight is given to order statistics whose expected values over all possible samples are just inside this region. This could be achieved by using an estimator involving a rejection criterion or an estimator, such as an M-estimator, for which the weights assigned to points depend on their position rather than on their rank in the ordered sample. Such estimators may be more efficient at separating the good points (to use) from the bad points.

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APPENDIX 1

THE NEAR-NORMAL DISTRIBUTIONS

This appendix gives the parameters of forty-eight near-normal distributions. Distribution (12,1) is an approximation to the normal distribution for which the expectations and covariances of the order statistics, and the best linear estimators and their variances, are given in section 2.5.

The densities are piecewise linear and the R^{th} segment extends from P [R] to P [R+1] and has equation Y = A [R] X + B [R].

The cutpoints are the points where the density cuts the normal density. The probability difference in a particular region is the probability mass for the normal distribution minus the probability mass for the given distribution in that region.

All the distributions are symmetric and have mean 0 and variance 1. Parameters not given can be found from considerations of symmetry.

DISTRIBUTION(1,1)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.00311070553	0.00229286203751	0.0114714426061
2	-3.0	0.0224489795901	0.07734693878
3	-2.51	0.09193988568	0.2517691131
4	-1.8	0.0	0.08627731883
5	-0.75	0.343574638302	0.60768097869
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.004592856
2	0.010000000	0.021000000
3	0.021000000	0.086277319
4	0.086277319	0.086277319
5	0.350000000	0.607680979

THE CUTPOINTS ARE -1.75, -0.75, 0.75 AND 1.75.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 1.75	BEYOND 1.75
-0.0858	0.1003	-0.0145

DISTRIBUTION(1,2)

THIS DISTRIBUTION IS DEFINED OVER 14 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-3.58388135678	0.0077438103586	0.0277528975746
2	-3.0	0.0225	0.0735
3	-2.6	0.067505492289	0.190514280001
4	-2.12	0.1050667093	0.270144059999
5	-1.41	0.225210084	0.4395462185
6	-0.815	0.6944220338	0.82195395689
7	-0.7	0.154194255272	0.443794511883
8	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.004521466
2	0.006000000	0.015000000
3	0.015000000	0.047402636
4	0.047402636	0.122000000
5	0.122000000	0.256000000
6	0.255999999	0.335858533
7	0.335858533	0.443794512

THE CUTPOINTS ARE -1.75, -0.75, 0.75 AND 1.75.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 1.75	BEYOND 1.75
-0.0154	0.0206	-0.0052

DISTRIBUTION(2,1)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.5084591476	0.00222491780042	0.0122558688105
2	-3.0	0.0250943349699	0.0852830049
3	-2.7	0.0	0.0175283004901
4	-1.5	0.233333333301	0.5
5	-0.6	0.124093301783	0.434455981051
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.005581115
2	0.010000000	0.017528300
3	0.017528300	0.017528300
4	0.150000000	0.360000000
5	0.360000000	0.434455981

THE CUTPOINTS ARE -2.50, -1.50, 1.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 1.50	1.50 TO 2.50	BEYOND 2.50
-0.0346	0.0431	-0.0084

DISTRIBUTION(2,2)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.85597686699	0.00255468615347	0.0124054968638
2	-3.0	0.01235849755	0.0484245443499
3	-2.3	0.0309734513301	0.09123893805
4	-1.735	0.391564236999	0.7168639511
5	-1.42	0.2164752885	0.4682376442
6	-0.5	0.140459547067	0.430229773534
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.004741438
2	0.011349052	0.020000000
3	0.020000000	0.037500000
4	0.037500000	0.160842735
5	0.160842735	0.360000000
6	0.360000000	0.430229774

THE CUTPOINTS ARE -2.50, -1.50, 1.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 1.50	1.50 TO 2.50	BEYOND 2.50
-0.0156	0.0210	-0.0054

DISTRIBUTION(3,1)

THIS DISTRIBUTION IS DEFINED OVER 8 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-6.2518474624	0.00151307219729	0.0094594965772
2	-3.0	0.0433232961 2	0.144969888301
3	-2.1	0.0	0.0539909665099
4	-0.25	7.9542648668	2.4385662167
5	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.004920280
2	0.015000000	0.053990966
3	0.053990967	0.053990967
4	0.450000000	2.438566217

THE CUTPOINTS ARE -2.00, -0.25, 0.25 AND 2.00.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.00	BEYOND 2.00
-0.2624	0.2841	-0.0217

DISTRIBUTION(3,2)

THIS DISTRIBUTION IS DEFINED OVER 18 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.0843079592	0.00425269966328	0.0173693350828
2	-3.0	0.0216666666699	0.07
3	-2.7	0.0462 5	0.136375
4	-2.3	0.0856820 2325	0.227068653599
5	-2.02	0.0	0.0539909665099
6	-1.91	0.151828996401	0.3439843496
7	-1.495	0.224120602999	0.452060301501
8	-0.5	0.168	0.424
9	-0.25	0.510052249374	0.527513062343
10	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.004611236
2	0.005000000	0.011500000
3	0.011500000	0.030000000
4	0.030000000	0.053990967
5	0.053990967	0.053990967
6	0.053990966	0.117000000
7	0.117000000	0.340000000
8	0.340000000	0.382000000
9	0.400000000	0.527513062

THE CUTPOINTS ARE -2.00, -0.25, 0.25 AND 2.00.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.00	BEYOND 2.00
-0.0172	0.0206	-0.0034

DISTRIBUTION(3,3)

THIS DISTRIBUTION IS DEFINED OVER 18 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-3.89537558593	0.00508920173164	0.0198243521772
2	-3.0	0.0216666666699	0.07
3	-2.7	0.04625	0.136375
4	-2.3	0.0856820 2325	0.227068653599
5	-2.02	0.0	0.0539909665099
6	-1.93	0.14895752 6	0.341478991701
7	-1.507	0.224572003999	0.455430010101
8	-0.514	0.159090909099	0.421772727301
9	-0.25	0.390335621662	0.497583905417
10	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.004556747
2	0.005000000	0.011500000
3	0.011500000	0.030000000
4	0.030000000	0.053990967
5	0.053990967	0.053990967
6	0.053990967	0.117000000
7	0.117000000	0.340000000
8	0.340000000	0.382000000
9	0.400000000	0.497583905

THE CUTPOINTS ARE -2.00, -0.25, 0.25 AND 2.00.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.00	BEYOND 2.00
-0.0135	0.0164	-0.0029

DISTRIBUTION(4,1)

THIS DISTRIBUTION IS DEFINED OVER 6 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-7.062001661	0.00169698435301	0.0119841063197
2	-3.0	0.0	0.0175283004901
3	-0.75	0.65444027083	0.84083020009
4	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.006893153
2	0.017528300	0.017528300
3	0.349999997	0.840830200

THE CUTPOINTS ARE -2.50, -0.75, 0.75 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.50	BEYOND 2.50
-0.1732	0.1897	-0.0166

DISTRIBUTION(4,2)

THIS DISTRIBUTION IS DEFINED OVER 8 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-7.5169551308	0.00117630431188	0.00884222673 26
2	-3.0	0.0067936 21988	0.03451235546
3	-0.885	2.01953653499	1.815789833
4	-0.73	0.61896963412	0.79337599494
5	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.005313314
2	0.014131489	0.028500000
3	0.028500000	0.341528162
4	0.341528162	0.793375995

THE CUTPOINTS ARE -2.50, -0.75, 0.75 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.50	BEYOND 2.50
-0.1473	0.1610	-0.0137

DISTRIBUTION(4,3)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-6.5200397159	0.00161411646724	0.0105241034725
2	-3.0	0.01505660098	0.05516980294
3	-2.0	0.0356334271701	0.09632345533
4	-1.3	0.456613513101	0.64359756709
5	-0.64	0.433974209005	0.62910841246
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.005681754
2	0.010000000	0.025056601
3	0.025056601	0.050000000
4	0.050000000	0.351364919
5	0.351364919	0.629108412

THE CUTPOINTS ARE -2.50, -0.75, 0.75 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.50	BEYOND 2.50
-0.0763	0.0869	-0.0107

DISTRIBUTION(4,4)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.7932577015	0.00179434531633	0.01039510482 3
2	-3.0	0.01505660098	0.05516980294
3	-2.2	0.06608418342	0.1674304843
4	-1.55	0.246666666701	0.447333333301
5	-0.8	1.022748644	1.06819891599
6	-0.7	0.232902132826	0.515306358198
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.005012069
2	0.010000000	0.022045281
3	0.022045281	0.065000000
4	0.065000000	0.250000000
5	0.250000001	0.352274865
6	0.352274865	0.515306358

THE CUTPOINTS ARE -2.50, -0.75, 0.75 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.50	BEYOND 2.50
-0.0466	0.0543	-0.0077

DISTRIBUTION(4,5)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.09119634854	0.00365872997511	0.0186273126897
2	-3.0	0.01505660098	0.05516980294
3	-2.2	0.06608418342	0.1674304843
4	-1.55	0.246666666701	0.447333333301
5	-0.8	1.022748644	1.06819891599
6	-0.7	0.228820500306	0.512449215385
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.007651123
2	0.010000000	0.022045281
3	0.022045281	0.065000000
4	0.065000000	0.250000000
5	0.250000001	0.352274865
6	0.352274865	0.512449215

THE CUTPOINTS ARE -2.50, -0.75, 0.75 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.50	BEYOND 2.50
-0.0456	0.0543	-0.0087

DISTRIBUTION(4,6)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.3624045552	0.0021501695 2936	0.0115300788786
2	-3.0	0.01505660098	0.05516980294
3	-2.2	0.079924532029	0.197879251 3
4	-1.6	0.2375	0.45
5	-0.8	0.822748644	0.918198916
6	-0.7	0.211682263247	0.490452449452
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.005079570
2	0.010000000	0.022045281
3	0.022045281	0.070000000
4	0.070000000	0.260000000
5	0.260000001	0.342274865
6	0.342274865	0.490452449

THE CUTPOINTS ARE -2.50, -0.75, 0.75 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.50	BEYOND 2.50
-0.0342	0.0408	-0.0067

DISTRIBUTION(4,7)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.76641058786	0.0026280312963	0.0125262761959
2	-3.0	0.01505660098	0.05516980294
3	-2.3	0.08378519894	0.213245578299
4	-1.65	0.2294117647	0.4535294118
5	-0.8	0.622748642	0.768198914
6	-0.7	0.182977989753	0.460359457456
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.004642182
2	0.010000000	0.020539621
3	0.020539621	0.075000000
4	0.075000000	0.270000000
5	0.270000000	0.332274865
6	0.332274865	0.460359457

THE CUTPOINTS ARE -2.50, -0.75, 0.75 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.50	BEYOND 2.50
-0.0199	0.0247	-0.0048

DISTRIBUTION(4,8)

THIS DISTRIBUTION IS DEFINED OVER 14 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-3.26749699751	0.0058354548822	0.0190673313068
2	-2.6	0.08094339902	0.219886798001
3	-2.0	0.15	0.358
4	-1.5	0.2415384615	0.495307692299
5	-0.85	0.111374322	0.384668174
6	-0.7	0.1776461713	0.431058468699
7	-0.4	0.056074103849	0.382429641541
8	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.003895149
2	0.009433961	0.058000000
3	0.058000000	0.133000000
4	0.133000000	0.290000000
5	0.290000000	0.306706149
6	0.306706149	0.360000000
7	0.360000000	0.382429642

THE CUTPOINTS ARE -2.50, -0.75, 0.75 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.50	BEYOND 2.50
0.0097	-0.0132	0.0036

DISTRIBUTION(5,1)

THIS DISTRIBUTION IS DEFINED OVER 6 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-12.428859793	1.25431897949E-4	0.00155897547317
2	-3.5	0.0	0.004431848412
3	-1.25	0.220836276485	0.526045345656
4	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.001119964
2	0.004431848	0.004431848
3	0.250000000	0.526045346

THE CUTPOINTS ARE -3.00, -1.25, 1.25 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 1.25	1.25 TO 3.00	BEYOND 3.00
-0.0907	0.0965	-0.0059

DISTRIBUTION(5,2)

THIS DISTRIBUTION IS DEFINED OVER 14 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.8135532817	0.00122974925624	0.0071492128243
2	-3.2	0.0	0.004431848412
3	-2.6	0.0618448842 2	0.165228547399
4	-1.75	0.2177777778	0.4381111111
5	-1.3	0.552981708	0.87387622
6	-1.2	0.228158328	0.484079164
7	-0.5	0.103729898037	0.421864949018
8	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.003214015
2	0.004431848	0.004431848
3	0.004431848	0.057000000
4	0.057000000	0.155000000
5	0.155000000	0.210298170
6	0.210289170	0.370000000
7	0.370000000	0.421864949

THE CUTPOINTS ARE -3.00, -1.25, 1.25 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 1.25	1.25 TO 3.00	BEYOND 3.00
-0.0165	0.0203	-0.0037

DISTRIBUTION(6,1)

THIS DISTRIBUTION IS DEFINED OVER 6 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-6.2945142308	0.00270623476334	0.0170344332298
2	-2.75	0.0	0.0317396518401
3	-0.25	8.916827852 6	2.72920696315
4	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.009592288
2	0.031739652	0.031739652
3	0.500000000	2.729206963

THE CUTPOINTS ARE -2.25, -0.25, 0.25 AND 2.25.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.25	BEYOND 2.25
-0.3049	0.3256	-0.0206

DISTRIBUTION(6,2)

THIS DISTRIBUTION IS DEFINED OVER 8 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.6586991371	0.00354587686709	0.0200650503682
2	-2.75	0.0246144473699	0.08712215843
3	-0.98	0.4433809819	0.497513362299
4	-0.23	5.6379274248	1.69225904468
5	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.010313889
2	0.019432428	0.063000000
3	0.063000000	0.395535736
4	0.395535737	1.692259045

THE CUTPOINTS ARE -2.25, -0.25, 0.25 AND 2.25.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.25	BEYOND 2.25
-0.1492	0.1648	-0.0156

DISTRIBUTION(6,3)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.879027523	0.0049703454176	0.0242504520913
2	-2.7	0.0330413926399	0.106082785 3
3	-2.0	0.06349206349	0.16698412 7
4	-1.37	0.2685185185	0.447870370401
5	-0.29	0.41670292	0.490843847499
6	-0.21	3.259006010 02	1.08772749637
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.010830519
2	0.016871025	0.040000000
3	0.040000000	0.080000000
4	0.080000000	0.370000000
5	0.370000001	0.403336234
6	0.403336234	1.087727496

THE CUTPOINTS ARE -2.25, -0.25, 0.25 AND 2.25.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.25	BEYOND 2.25
-0.0737	0.0842	-0.0105

DISTRIBUTION(6,4)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.53563018303	0.00623230808 59	0.0282674446644
2	-2.7	0.0330413926399	0.1060827853
3	-2.0	0.0818181818 2	0.2036363636
4	-1.45	0.245689655199	0.44125
5	-0.29	0.4167029 2	0.490843847499
6	-0.21	2.52544592477	0.93367987848
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.011440213
2	0.016871025	0.040000000
3	0.040000000	0.085000000
4	0.085000000	0.370000000
5	0.370000001	0.403336234
6	0.403336234	0.933679878

THE CUTPOINTS ARE -2.25, -0.25, 0.25 AND 2.25.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.25	BEYOND 2.25
-0.0575	0.0667	-0.0092

DISTRIBUTION(6,5)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-3.92125571375	0.0100572221017	0.0394369396306
2	-2.7	0.0417356544001	0.125644874301
3	-2.1	0.1018867925	0.251962264199
4	-1.57	0.219083969499	0.4359618321
5	-0.26	0.76681168	0.57837104
6	-0.23	1.13729115656	0.66358131963
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.012282440
2	0.012958607	0.038000000
3	0.038000000	0.092000000
4	0.092000000	0.379000000
5	0.379000003	0.402004354
6	0.402004354	0.663581320

THE CUTPOINTS ARE -2.25, -0.25, 0.25 AND 2.25.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.25	BEYOND 2.25
-0.0317	0.0371	-0.0053

DISTRIBUTION(6,6)

THIS DISTRIBUTION IS DEFINED OVER 14 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-3.47781022292	0.0145457251929	0.0505872717754
2	-2.7	0.0205486196401	0.077974046
3	-2.14	0.1111111111	0.2717777778
4	-1.6	0.2328358209	0.466537313399
5	-0.93	0.1985185185	0.434622222199
6	-0.255	0.533623359999	0.520073956
7	-0.23	0.56150972288	0.526487819465
8	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.011313814
2	0.022492773	0.034000000
3	0.034000000	0.094000000
4	0.094000000	0.250000000
5	0.250000000	0.384000000
6	0.383999999	0.397340583
7	0.397340583	0.526487819

THE CUTPOINTS ARE -2.25, -0.25, 0.25 AND 2.25.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.25	BEYOND 2.25
-0.0154	0.0198	-0.0044

DISTRIBUTION(7,1)

THIS DISTRIBUTION IS DEFINED OVER 6 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-8.5652377164	8.0718404978E-4	0.0069137232539
2	-2.99999999873	0.0	0.0175283004976
3	-0.499999999789	1.94943399187	1.37471699569
4	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.004492171
2	0.017528300	0.017528300
3	0.400000000	1.374716996

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
-0.2522	0.2673	-0.0151

DISTRIBUTION(7,2)

THIS DISTRIBUTION IS DEFINED OVER 4 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.30022765737	0.0	0.0175283000647
2	-0.500000012134	0.0	0.86677691796
3	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.017528300	0.017528300
2	0.866776918	0.866776918

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
-0.2419	0.2673	-0.0253

DISTRIBUTION(7,3)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-7.0366975108	8.9818609004E-4	0.00632026382 49
2	-3.69999999886	0.00247169950151	0.0237075492573
3	-1.49999999955	0.200000000123	0.320000000098
4	-0.59999999981	1.52065326994	1.11239196234
5	-0.399999999878	1.07395232466	0.93371158368
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.002996975
2	0.014562261	0.020000000
3	0.020000000	0.200000000
4	0.200000001	0.504130655
5	0.504130654	0.933711584

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
-0.1389	0.1570	-0.0180

DISTRIBUTION(7,4)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.93463690036	0.0091843733754	0.045321547746
2	-3.69999999952	0.00247169950066	0.0237075492532
3	-1.49999999981	0.200000000053	0.320000000042
4	-0.59999999992	1.5206532694	1.11239196215
5	-0.399999999947	1.04546118528	0.92301335561
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.011339366
2	0.014562261	0.020000000
3	0.020000000	0.200000000
4	0.200000001	0.504130654
5	0.504828882	0.923013356

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
-0.1369	0.1570	-0.0200

DISTRIBUTION(7,5)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.9999999987	0.00106521739147	0.0063913043474
2	-3.6999999992	0.00460691708102	0.0290455931963
3	-1.79999999961	0.18290495678	0.349982064676
4	-0.81999999982	0.475204145907	0.58966739963
5	-0.399999999912	0.66818006028	0.71359843516
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.002450000
2	0.012000000	0.020753142
3	0.020753143	0.200000000
4	0.200000000	0.399585741
5	0.446326411	0.713598435

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
-0.0781	0.0924	-0.0143

DISTRIBUTION(7,6)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.53436364954	0.00475730966798	0.0215713720122
2	-2.99999999969	0.0082389983766	0.0381257964339
3	-2.19999999978	0.11250000023	0.267500000027
4	-1.39999999985	0.268961474254	0.486546063948
5	-0.39999999996	0.412139408665	0.543817237667
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.007299443
2	0.013408801	0.020000000
3	0.020000000	0.110000000
4	0.110000000	0.378961474
5	0.378961474	0.543817238

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
-0.0296	0.0368	-0.0071

DISTRIBUTION(7,7)

THIS DISTRIBUTION IS DEFINED OVER 14 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.44360814214	0.00308060759721	0.0136890130017
2	-3.0	0.0190566010001	0.065169803
3	-2.4	0.06792453202	0.1824528374
4	-1.95	0.142857142901	0.328571428599
5	-1.46	0.2326315789	0.459642105299
6	-0.51	1.10653268	0.90533167
7	-0.49	0.227712138443	0.474709604625
8	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.004447190
2	0.008000000	0.019433961
3	0.019433961	0.050000000
4	0.050000000	0.120000000
5	0.120000000	0.341000000
6	0.341000003	0.363130657
7	0.363130657	0.474709605

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
-0.0174	0.0208	-0.0034

DISTRIBUTION(7,8)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-3.39516575326	0.00484980114723	0.0164658787687
2	-2.5600000002	0.123584975481	0.326490738975
3	-2.48000000019	0.084745762696	0.230169491482
4	-1.89000000014	0.168749999973	0.388937499969
5	-1.41000000012	0.236144578264	0.483963855362
6	-0.58000000004	0.054566585 2933	0.379348618946
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.004050388
2	0.010113202	0.020000000
3	0.020000000	0.070000000
4	0.070000000	0.151000000
5	0.151000000	0.347000000
6	0.347699999	0.379348619

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
0.0086	-0.0123	0.0037

DISTRIBUTION(7,9)

THIS DISTRIBUTION IS DEFINED OVER 8 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-2.50000000009	0.059393027625	0.174809081394
2	-2.09	0.117948717892	0.298512820489
3	-1.7	0.224836572384	0.480222173084
4	-0.57000000001	-4.14857232358E-11	0.352065326764
5	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.026326512	0.050677654
2	0.052000000	0.098000000
3	0.098000000	0.352065327
4	0.352065327	0.352065327

THE CUTPOINTS ARE -2.50, -0.50, 0.50 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.50	0.50 TO 2.50	BEYOND 2.50
0.0154	-0.0216	0.0062

DISTRIBUTION(8,1)

THIS DISTRIBUTION IS DEFINED OVER 6 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-10.0225256315	4.36042359034E-4	0.00437024572071
2	-3.24999999726	0.0	0.00909356225096
3	-0.74999999936	0.72805722321	0.89604291732
4	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.002953108
2	0.009093563	0.009093563
3	0.350000000	0.896042917

THE CUTPOINTS ARE -2.75, -0.75, 0.75 AND 2.75.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.75	BEYOND 2.75
-0.1939	0.2055	-0.0116

DISTRIBUTION(8,2)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-9.1084215784	5.8273184874E-4	0.00530776734374
2	-3.25000000016	0.0	0.00909356225015
3	-2.25000000012	0.0154862499985	0.0439376249979
4	-0.90000000004	1.80758288082	1.65682459291
5	-0.70000000003	0.56451029372	0.78667378159
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.003413889
2	0.009093563	0.009093563
3	0.009093562	0.030000000
4	0.030000000	0.391516576
5	0.391516576	0.786673782

THE CUTPOINTS ARE -2.75, -0.75, 0.75 AND 2.75.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.75	BEYOND 2.75
-0.1563	0.1679	-0.0116

DISTRIBUTION(8,3)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-8.8609879896	5.08207773844E-4	0.00450322298221
2	-3.24999999985	0.0	0.0090935625004
3	-2.2499999999	0.0389585119035	0.096750214294
4	-1.2	0.55808318275	0.71969981912
5	-0.64999999996	0.511041411532	0.68912266778
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.002851548
2	0.009093563	0.009093563
3	0.009093563	0.050000000
4	0.050000000	0.356945750
5	0.356945750	0.689122668

THE CUTPOINTS ARE -2.75, -0.75, 0.75 AND 2.75.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.75	BEYOND 2.75
-0.0995	0.1091	-0.0096

DISTRIBUTION(8,4)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-8.073618997	6.017028148E-4	0.00485791927218
2	-3.2499999992	0.0	0.0090935625042
3	-2.24999999945	0.064112039501	0.153345651439
4	-1.29999999968	0.420249876908	0.61632483995
5	-0.59999999984	0.421504595182	0.61707767092
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.002902385
2	0.009093563	0.009093563
3	0.009093563	0.070000000
4	0.070000000	0.364174914
5	0.364174914	0.617077671

THE CUTPOINTS ARE -2.75, -0.75, 0.75 AND 2.75.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.75	BEYOND 2.75
-0.0709	0.0795	-0.0086

DISTRIBUTION(8,5)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-6.1908019464	0.00134154713675	0.008305252626256
2	-3.2	0.0	0.00909356 2502
3	-2.3	0.08863304688	0.212949570399
4	-1.5	0.2948499096	0.522274864399
5	-0.6	0.4337872003	0.56140975225
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.004012302
2	0.009093563	0.009093563
3	0.009093563	0.080000000
4	0.080000000	0.345364919
5	0.301137432	0.561409752

THE CUTPOINTS ARE -2.75, -0.75, 0.75 AND 2.75.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.75	BEYOND 2.75
-0.0339	0.0410	-0.0071

DISTRIBUTION(8,6)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.1754488752 6	0.00167503174221	0.0086690411462
2	-3.1	0.0	0.00909356 2502
3	-2.4	0.1011330469	0.2518128 75
4	-1.6	0.2409638554	0.4755421687
5	-0.77	0.55687161	0.71879114
6	-0.72	0.187677561553	0.4529714 25111
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.003476443
2	0.009093563	0.009093563
3	0.009093562	0.090000000
4	0.090000000	0.290000000
5	0.290000000	0.317843581
6	0.317843581	0.452971425

THE CUTPOINTS ARE -2.75, -0.75, 0.75 AND 2.75.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 2.75	BEYOND 2.75
-0.0134	0.0172	-0.0038

DISTRIBUTION(9,1)

THIS DISTRIBUTION IS DEFINED OVER 6 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-8.4760705075	8.9063215783E-4	0.0075490609639
2	-3.49999999817	0.0	0.00443184841433
3	-0.99999999948	0.357219593934	0.656503581
4	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.004431848
2	0.004431848	0.004431848
3	0.299283987	0.656503581

THE CUTPOINTS ARE -3.00, -1.00, 1.00 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 1.00	1.00 TO 3.00	BEYOND 3.00
-0.1365	0.1484	-0.0119

DISTRIBUTION(9,2)

THIS DISTRIBUTION IS DEFINED OVER 10 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-8.916828654	4.77131224891E-4	0.00425449737853
2	-3.50000000041	0.0	0.0044318484095
3	-2.5000000029	0.0296401263231	0.0785321642l1
4	-1.30000000015	0.67323574864	0.91520647339
5	-0.9000000001	0.280704476754	0.56192832854
6	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.002584538
2	0.004431848	0.004431848
3	0.004431848	0.040000000
4	0.040000000	0.309294300
5	0.309294299	0.561928329

THE CUTPOINTS ARE -3.00, -1.00, 1.00 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 1.00	1.00 TO 3.00	BEYOND 3.00
-0.0783	0.0861	-0.0079

DISTRIBUTION(9,3)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-6.7032199147	0.0011695202211	0.0078395512333
2	-3.49999999985	0.0	0.0044318484122
3	-2.2999999999	0.094460189498	0.221690284308
4	-1.5	0.25000000022	0.45500000002
5	-1.1	0.61970724505	0.86167797004
6	-0.89999999996	0.19991998 2671	0.483869433869
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.003746230
2	0.004431848	0.004431848
3	0.004431848	0.080000000
4	0.080000000	0.180000000
5	0.180000001	0.303941450
6	0.303941449	0.483869434

THE CUTPOINTS ARE -3.00, -1.00, 1.00 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 1.00	1.00 TO 3.00	BEYOND 3.00
-0.0405	0.0473	-0.0069

DISTRIBUTION(9,4)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-7.8566590477	3.16114271619E-4	0.00248360205308
2	-3.4999999999	0.0	0.00443184841214
3	-2.4	0.093668787995	0.229236939607
4	-1.7	0.215384615412	0.436153846213
5	-1.05	0.63941448673	0.88138521132
6	-0.89999999997	0.150764076743	0.441599842054
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.001377202
2	0.004431848	0.004431848
3	0.004431848	0.070000000
4	0.070000000	0.210000000
5	0.210000000	0.305912173
6	0.305912173	0.441599842

THE CUTPOINTS ARE -3.00, -1.00, 1.00 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 1.00	1.00 TO 3.00	BEYOND 3.00
-0.0224	0.0263	-0.0039

DISTRIBUTION(10,1)

THIS DISTRIBUTION IS DEFINED OVER 6 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-7.7978615881	0.00130324426297	0.0101625183782
2	-3.0	0.0	0.0175283004901
3	-0.25	9.97750956	2.99437739002
4	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.006252786
2	0.017528300	0.017528300
3	0.500000000	2.994377390

THE CUTPOINTS ARE -2.50, -0.25, 0.25 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.50	BEYOND 2.50
-0.3381	0.3556	-0.0176

DISTRIBUTION(10,2)

THIS DISTRIBUTION IS DEFINED OVER 12 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-3.71495581127	0.0082165762319	0.0305242176216
2	-3.0	0.01123499777	0.04561579491
3	-2.28	0.115	0.2822
4	-1.68	0.2070175439	0.436789473701
5	-0.255	0.5336233599999	0.520073956
6	-0.23	0.58769451978	0.532510322755
7	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.005874489
2	0.011910802	0.020000000
3	0.020000000	0.089000000
4	0.089000000	0.384000000
5	0.383999999	0.397340583
6	0.397340583	0.532510323

THE CUTPOINTS ARE -2.50, -0.25, 0.25 AND 2.50.

THE PROBABILITY DIFFERENCES

0 TO 0.25	0.25 TO 2.50	BEYOND 2.50
-0.0161	0.0193	-0.0033

DISTRIBUTION(11,1)

THIS DISTRIBUTION IS DEFINED OVER 6 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-10.5081498945	4.07214497855E-4	0.00427907709827
2	-3.5	0.0	0.004431848411
3	-0.75	0.76555525996	0.92416644497
4	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.002853820
2	0.004431848	0.004431848
3	0.350000000	0.924166445

THE CUTPOINTS ARE -3.00, -0.75, 0.75 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 3.00	BEYOND 3.00
-0.2044	0.2153	-0.0109

DISTRIBUTION(11,2)

THIS DISTRIBUTION IS DEFINED OVER 14 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-7.0573541043	3.16086390018E-4	0.00223073358194
2	-3.5	0.0	0.004431848412
3	-2.7	0.05011956419	0.1397546717
4	-2.13	0.1056074766	0.2579439252
5	-1.595	0.239393939401	0.4713333333
6	-0.77	0.70687161	0.83129114
7	-0.72	0.186691654219	0.45676157187
8	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.001124431
2	0.004431848	0.004431848
3	0.004431848	0.033000000
4	0.033000000	0.089500000
5	0.089500000	0.287000000
6	0.287000000	0.322343581
7	0.322343581	0.456761572

THE CUTPOINTS ARE -3.00, -0.75, 0.75 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 3.00	BEYOND 3.00
-0.0165	0.0193	-0.0029

DISTRIBUTION(11,3)

THIS DISTRIBUTION IS DEFINED OVER 14 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-5.9838787941	6.48334589E-4	0.0038795555986 2
2	-3.5	0.0	0.004431848412
3	-2.7	0.05011956419	0.1397546717
4	-2.13	0.1115384615	0.2705769231
5	-1.61	0.236904761899	0.4724166667
6	-0.77	0.55687161	0.71879114
7	-0.72	0.183190496437	0.449740738268
8	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	0.000000000	0.001610385
2	0.004431848	0.004431848
3	0.004431848	0.033000000
4	0.033000000	0.091000000
5	0.091000000	0.290000000
6	0.290000000	0.317843581
7	0.317843581	0.449740738

THE CUTPOINTS ARE -3.00, -0.75, 0.75 AND 3.00.

THE PROBABILITY DIFFERENCES

0 TO 0.75	0.75 TO 3.00	BEYOND 3.00
-0.0122	0.0151	-0.0029

DISTRIBUTION(12,1)

THIS DISTRIBUTION IS DEFINED OVER 14 REGIONS.

THE PARAMETERS OF THE DISTRIBUTION

R	P[R]	A[R]	B[R]
1	-4.68363603266	1.35639704009E-4	6.3528700507E-4
2	-3.99939598088	0.001410413 27633	0.005690821 2723
3	-3.5001785904	0.006886011956	0.0246146369684
4	-2.99954698591	0.0256296321525	0.080199036531
5	-2.50032959544	0.07887357071	0.211030181828
6	-1.90070304026	0.213558391022	0.456036224121
7	-0.35001785904	0.072550753768	0.403640943487
8	0.0		

R	DENSITY AT LEFT END OF RTH REGION	DENSITY AT RIGHT END OF RTH REGION
1	-0.000000000	0.000092810
2	0.000050020	0.000754123
3	0.000512365	0.003959721
4	0.003321801	0.016116559
5	0.013820259	0.061114946
6	0.050125141	0.381286973
7	0.378246884	0.403640943

APPENDIX 2

THE COEFFICIENTS AND VARIANCES OF THE BEST LINEAR LOCATION ESTIMATORS FOR THE NEAR-NORMAL DISTRIBUTIONS FOR SAMPLE SIZES OF 4, 8 AND 16.

DISTRIBUTION (1, 1)

CUTPOINTS: 0.75 AND 1.75. PROBABILITY DIFFERENCE: 0.1003.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.048720655	0.007492709	0.014329498
2	0.451279345	-0.018082833	-0.020418069
3		0.144393459	-0.041672815
4		0.366196665	-0.001498020
5			0.103657974
6			0.171319821
7			0.149828845
8			0.124452767

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.209792547	0.087562187	0.038609817

DISTRIBUTION (1, 2)

CUTPOINTS: 0.75 AND 1.75. PROBABILITY DIFFERENCE: 0.0206.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.205412307	0.097836202	0.064128458
2	0.294587693	0.081563178	0.019497811
3		0.145962922	0.025644188
4		0.174637698	0.055498593
5			0.081098326
6			0.087325083
7			0.084198355
8			0.082609187

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.248440162	0.123201076	0.060982362

DISTRIBUTION (2, 1)

CUTPOINTS: 1.50 AND 2.50. PROBABILITY DIFFERENCE: 0.0431.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.129124870	0.005859833	-0.019602848
2	0.370875130	0.224693141	0.045578542
3		0.169159783	0.182435353
4		0.100287243	0.104550339
5			0.050364423
6			0.042369378
7			0.045197893
8			0.049106920

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.235596663	0.111015317	0.052239171

DISTRIBUTION (2, 2)

CUTPOINTS: 1.50 AND 2.50.PROBABILITY DIFFERENCE: 0.0210.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.181906144	0.044215805	-0.004704149
2	0.318093856	0.191158540	0.071941514
3		0.142973240	0.129846695
4		0.121652415	0.074721741
5			0.049894329
6			0.051218359
7			0.059891218
8			0.067190294

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.245999381	0.120414461	0.058735054

DISTRIBUTION (3, 1)

CUTPOINTS: 0.25 AND 2.00.PROBABILITY DIFFERENCE: 0.2841.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.028289536	-0.007035506	-0.001001852
2	0.528289536	-0.025810045	-0.007323563
3		-0.032001239	-0.011026631
4		0.564846791	-0.014712595
5			-0.017448695
6			0.005182215
7			0.157102108
8			0.389229015

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.112111138	0.014501004	0.002687998

DISTRIBUTION (3, 2)

CUTPOINTS: 0.25 AND 2.00.PROBABILITY DIFFERENCE: 0.0206.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.194380385	0.084746266	0.039411217
2	0.305619615	0.095668033	0.045821647
3		0.125139063	0.050704391
4		0.194446638	0.040711000
5			0.031892272
6			0.047476578
7			0.096213908
8			0.147768987

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.247522479	0.122440159	0.060353214

DISTRIBUTION (3, 3)

CUTPOINTS: 0.25 AND 2.00. PROBABILITY DIFFERENCE: 0.0164.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.209002401	0.096514067	0.048043352
2	0.290997599	0.099524028	0.047003753
3		0.125213474	0.052859485
4		0.178748431	0.045167792
5			0.038320562
6			0.051183107
7			0.089260463
8			0.128161486

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.248673296	0.123559316	0.061246560

DISTRIBUTION (4, 1)

CUTPOINTS: 0.75 AND 2.50. PROBABILITY DIFFERENCE: 0.1897.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.028479140	-0.020831799	-0.010086251
2	0.528479140	0.003658188	-0.020996751
3		0.255810525	0.005592759
4		0.261363086	0.139173684
5			0.162699087
6			0.077654857
7			0.067076720
8			0.078885894

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.103062392	0.038679307	0.017112196

DISTRIBUTION (4, 2)

CUTPOINTS: 0.75 AND 2.50. PROBABILITY DIFFERENCE: 0.1610.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.025912257	-0.021640016	-0.010182911
2	0.525912257	0.010960072	-0.023679051
3		0.256470939	0.009835249
4		0.254209005	0.137553194
5			0.141527793
6			0.078597118
7			0.076021300
8			0.090327308

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.113995982	0.044906816	0.020203412

DISTRIBUTION (4, 3)

CUTPOINTS: 0.75 AND 2.50. PROBABILITY DIFFERENCE: 0.0869.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.008543548	-0.021220112	-0.012661837
2	0.491456452	0.059968420	-0.018825954
3		0.224047503	0.042214180
4		0.237204189	0.103630732
5			0.088492061
6			0.079564204
7			0.098103238
8			0.119483377

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.174007419	0.076449043	0.035947636

DISTRIBUTION (4, 4)

CUTPOINTS: 0.75 AND 2.50. PROBABILITY DIFFERENCE: 0.0543.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.071722911	-0.003828350	-0.009477753
2	0.428277089	0.104242912	0.004604799
3		0.196762597	0.050587529
4		0.202822841	0.084450359
5			0.096505758
6			0.092429383
7			0.089211959
8			0.091687967

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.217355858	0.101468116	0.048971568

DISTRIBUTION (4, 5)

CUTPOINTS: 0.75 AND 2.50. PROBABILITY DIFFERENCE: 0.0543.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.075925136	-0.001950517	-0.008109044
2	0.424074864	0.098152435	0.000207631
3		0.198986989	0.049078734
4		0.204811093	0.085955463
5			0.098237151
6			0.093562635
7			0.089564015
8			0.091503415

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.219728915	0.102719808	0.049533476

DISTRIBUTION (4, 6)

CUTPOINTS: 0.75 AND 2.50.PROBABILITY DIFFERENCE: 0.0408.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.105470035	0.010778404	-0.005754036
2	0.394529965	0.120260180	0.020447508
3		0.179745190	0.060370188
4		0.189216226	0.076097946
5			0.083867620
6			0.085516571
7			0.087548213
8			0.091905990

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.230217239	0.109818269	0.053558321

DISTRIBUTION (4, 7)

CUTPOINTS: 0.75 AND 2.50.PROBABILITY DIFFERENCE: 0.0247.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.158340335	0.042869163	0.008143338
2	0.341659665	0.128976883	0.039592662
3		0.158660873	0.064390563
4		0.169493081	0.069105464
5			0.073081276
6			0.076694648
7			0.081723078
8			0.087268970

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.242801209	0.118895059	0.058720426

DISTRIBUTION (4, 8)

CUTPOINTS: 0.75 AND 2.50.PROBABILITY DIFFERENCE: -0.0132.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.298107071	0.184252533	0.121978196
2	0.201892929	0.110291780	0.053150736
3		0.106778483	0.061240046
4		0.098677203	0.061800245
5			0.055056675
6			0.050039361
7			0.048449834
8			0.048284908

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.248272043	0.123138339	0.061066814

DISTRIBUTION (5, 1)

CUTPOINTS: 1.25 AND 3.00.PROBABILITY DIFFERENCE: 0.0965.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.037344885	-0.011707309	-0.013356181
2	0.462655115	0.269694995	0.058346165
3		0.161424255	0.264150134
4		0.080588058	0.058899629
5			0.018182604
6			0.025605712
7			0.038958117
8			0.049213820

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.171334849	0.074509946	0.033171102

DISTRIBUTION (5, 2)

CUTPOINTS: 1.25 AND 3.00.PROBABILITY DIFFERENCE: 0.0203.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.169061969	0.034496910	-0.007210812
2	0.330938031	0.201242574	0.076619405
3		0.148192533	0.114568030
4		0.116067982	0.086915435
5			0.064110581
6			0.055490610
7			0.054318551
8			0.055188201

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.244109531	0.118549569	0.057666994

DISTRIBUTION (6, 1)

CUTPOINTS: 0.25 AND 2.25.PROBABILITY DIFFERENCE: 0.3256.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.030597256	-0.007074000	-0.002166838
2	0.530597256	-0.018812441	-0.006557763
3		-0.019309456	-0.009472413
4		0.545195896	-0.011259975
5			-0.001335034
6			0.087198968
7			0.229467109
8			0.214125947

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.073007423	0.006945642	0.001941941

DISTRIBUTION (6, 2)

CUTPOINTS: 0.25 AND 2.25.PROBABILITY DIFFERENCE: 0.1648.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.034757853	-0.011422413	-0.001738117
2	0.534757853	-0.041218842	-0.008180473
3		0.002675674	-0.015401222
4		0.549965580	-0.026831152
5			-0.039151111
6			-0.013527506
7			0.149933548
8			0.454896034

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.126122010	0.031403095	0.009204104

DISTRIBUTION (6, 3)

CUTPOINTS: 0.25 AND 2.25.PROBABILITY DIFFERENCE: 0.0842.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.029188964	-0.002079954	-0.000281833
2	0.470811036	-0.001334922	-0.003179404
3		0.089666362	0.003898707
4		0.413748515	-0.007367427
5			-0.030612128
6			-0.008848535
7			0.145242084
8			0.401148536

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.198165351	0.080786004	0.032066442

DISTRIBUTION (6, 4)

CUTPOINTS: 0.25 AND 2.25.PROBABILITY DIFFERENCE: 0.0667.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.064610893	0.010305576	0.003349419
2	0.435389107	0.023549015	0.004542694
3		0.105858317	0.017800826
4		0.360287092	0.003274472
5			-0.021596414
6			0.001305116
7			0.140778546
8			0.350545342

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.216747957	0.095331164	0.040866174

DISTRIBUTION (6, 5)

CUTPOINTS: 0.25 AND 2.25.PROBABILITY DIFFERENCE: 0.0371.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.147533072	0.054431379	0.023006429
2	0.352466928	0.069378891	0.030336944
3		0.118954699	0.041007954
4		0.257235030	0.022347310
5			0.004980247
6			0.027930833
7			0.119375991
8			0.231014293

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.241186490	0.116167081	0.055182202

DISTRIBUTION (6, 6)

CUTPOINTS: 0.25 AND 2.25.PROBABILITY DIFFERENCE: 0.0198.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.203946124	0.093479245	0.048821484
2	0.296053876	0.094947391	0.036433526
3		0.126044932	0.058903140
4		0.185528431	0.046323945
5			0.031671958
6			0.044304447
7			0.091535175
8			0.142006325

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.248325087	0.123168982	0.060860855

DISTRIBUTION (7, 1)

CUTPOINTS: 0.50 AND 2.50.PROBABILITY DIFFERENCE: 0.2673.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.031061407	-0.012428966	-0.005257472
2	0.531061407	-0.019032948	-0.013461427
3		0.143906243	-0.011724672
4		0.387555671	0.048530111
5			0.166322721
6			0.124315048
7			0.091296785
8			0.099978905

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.060508558	0.016077885	0.006933504

DISTRIBUTION (7, 2)

CUTPOINTS: 0.50 AND 2.50. PROBABILITY DIFFERENCE: 0.2673.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.030021673	-0.018974906	-0.008737025
2	0.530021673	-0.017399485	-0.016287395
3		0.180373054	-0.013527676
4		0.356001337	0.055963348
5			0.264808223
6			0.179701821
7			0.033992133
8			0.004086571

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.097773707	0.028203228	0.011096125

DISTRIBUTION (7, 3)

CUTPOINTS: 0.50 AND 2.50. PROBABILITY DIFFERENCE: 0.1570.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.030998132	-0.018175131	-0.006316632
2	0.530998132	-0.027295604	-0.017648801
3		0.129727546	-0.020758455
4		0.415743188	0.001741759
5			0.064531653
6			0.128418945
7			0.165701589
8			0.184329943

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.125824336	0.043027177	0.018471795

DISTRIBUTION (7, 4)

CUTPOINTS: 0.50 AND 2.50. PROBABILITY DIFFERENCE: 0.1570.

THE COEFFICIENTS, W[I], FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.030237701	-0.018858519	-0.006644881
2	0.530237701	-0.029404683	-0.017887819
3		0.124739049	-0.021191312
4		0.423524153	0.000156092
5			0.063978674
6			0.129820630
7			0.167098673
8			0.184669942

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.132014824	0.044142925	0.018831325

DISTRIBUTION (7, 5)

CUTPOINTS: 0.50 AND 2.50.PROBABILITY DIFFERENCE: 0.0924.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.012333254	-0.013509435	-0.006615003
2	0.487666746	0.012363102	-0.011012048
3		0.149523880	0.005732323
4		0.351622453	0.011103449
5			0.031805949
6			0.087517667
7			0.162519365
8			0.218948299

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.184098756	0.076989058	0.034662097

DISTRIBUTION (7, 6)

CUTPOINTS: 0.50 AND 2.50.PROBABILITY DIFFERENCE: 0.0368.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.120156172	0.026481792	0.004271298
2	0.379843828	0.092622816	0.023295565
3		0.152582531	0.049779533
4		0.228312861	0.047484973
5			0.044216613
6			0.063844741
7			0.110956662
8			0.156150616

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.234941807	0.112596957	0.054706436

DISTRIBUTION (7, 7)

CUTPOINTS: 0.50 AND 2.50.PROBABILITY DIFFERENCE: 0.0208.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.178633424	0.065386976	0.022946222
2	0.321366576	0.110420132	0.048953406
3		0.137939889	0.053708862
4		0.186253003	0.046442769
5			0.048743605
6			0.066981885
7			0.095149071
8			0.117074180

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.245805879	0.121240460	0.059976248

DISTRIBUTION (7, 8)

CUTPOINTS: 0.50 AND 2.50.PROBABILITY DIFFERENCE: -0.0123.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.292650860	0.176851440	0.117094769
2	0.207349140	0.111192055	0.049178031
3		0.113335091	0.056160233
4		0.098621415	0.065868496
5			0.066273253
6			0.057814452
7			0.047102975
8			0.040507792

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.248638186	0.123545371	0.061317001

DISTRIBUTION (7, 9)

CUTPOINTS: 0.50 AND 2.50.PROBABILITY DIFFERENCE: -0.0216.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.322642320	0.211760777	0.154251623
2	0.177357680	0.109990789	0.040870013
3		0.103449585	0.062257081
4		0.074798849	0.068981877
5			0.065594439
6			0.051730327
7			0.033986692
8			0.022327949

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.246089226	0.120825668	0.059204399

DISTRIBUTION (8, 1)

CUTPOINTS: 0.75 AND 2.75.PROBABILITY DIFFERENCE: 0.2055.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.021148339	-0.016292494	-0.008706262
2	0.521148339	0.036152444	-0.014159792
3		0.300581757	0.057128352
4		0.179558292	0.200299235
5			0.090098435
6			0.045526835
7			0.057644700
8			0.072168497

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.078976730	0.031879779	0.014248680

DISTRIBUTION (8, 2)

CUTPOINTS: 0.75 AND 2.75. PROBABILITY DIFFERENCE: 0.1679.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.021164644	-0.019781405	-0.010252444
2	0.521164644	0.041688196	-0.019826426
3		0.290977659	0.052574870
4		0.187115550	0.187719883
5			0.101540685
6			0.052776688
7			0.060711092
8			0.074755652

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.101361993	0.041990295	0.018919842

DISTRIBUTION (8, 3)

CUTPOINTS: 0.75 AND 2.75. PROBABILITY DIFFERENCE: 0.1091.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.007719435	-0.021042665	-0.011760152
2	0.507719435	0.066063963	-0.016217492
3		0.235933882	0.058718495
4		0.219044821	0.112863611
5			0.081064867
6			0.072116262
7			0.091226628
8			0.111987781

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.141646704	0.061715787	0.029002343

DISTRIBUTION (8, 4)

CUTPOINTS: 0.75 AND 2.75. PROBABILITY DIFFERENCE: 0.0795.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.015350157	-0.018835319	-0.012974554
2	0.484649843	0.090772494	-0.005558179
3		0.208316013	0.064062395
4		0.219746811	0.088676731
5			0.072364937
6			0.074565649
7			0.097947761
8			0.120915261

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.174977712	0.078505496	0.037403301

DISTRIBUTION (8, 5)

CUTPOINTS: 0.75 AND 2.75.PROBABILITY DIFFERENCE: 0.0410.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.081819891	0.000393487	-0.012297994
2	0.418180109	0.128966420	0.031396101
3		0.159512439	0.083915608
4		0.211127654	0.057074070
5			0.034520266
6			0.050667362
7			0.102283500
8			0.152441087

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.220742414	0.103944479	0.050135018

DISTRIBUTION (8, 6)

CUTPOINTS: 0.75 AND 2.75.PROBABILITY DIFFERENCE: 0.0172.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.174309976	0.050195687	0.005931767
2	0.325690024	0.150889127	0.066708273
3		0.144440420	0.074080820
4		0.154474767	0.064704379
5			0.061331583
6			0.065050754
7			0.075842152
8			0.086350273

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.245101027	0.120617016	0.059598005

DISTRIBUTION (9, 1)

CUTPOINTS: 1.00 AND 3.00.PROBABILITY DIFFERENCE: 0.1484.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.003660617	-0.018988736	-0.012763039
2	0.503660617	0.116783154	-0.004125383
3		0.286027737	0.156638560
4		0.116177845	0.183269308
5			0.042317367
6			0.031477515
7			0.045620924
8			0.057564748

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.123618583	0.053816157	0.023784953

DISTRIBUTION (9, 2)

CUTPOINTS: 1.00 AND 3.00.PROBABILITY DIFFERENCE: 0.0861.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.020598028	-0.020293446	-0.015692406
2	0.479401972	0.165892569	0.001443844
3		0.224768014	0.163062481
4		0.129632862	0.131960068
5			0.052682420
6			0.041876561
7			0.055534481
8			0.069132553

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.170127801	0.076253950	0.035331363

DISTRIBUTION (9, 3)

CUTPOINTS: 1.00 AND 3.00.PROBABILITY DIFFERENCE: 0.0473.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.082000703	-0.006562072	-0.016416017
2	0.417999297	0.185431417	0.031017049
3		0.185347948	0.122621432
4		0.135782707	0.109646598
5			0.071613273
6			0.053881276
7			0.058756007
8			0.068880381

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.218702867	0.101988621	0.048851471

DISTRIBUTION (9, 4)

CUTPOINTS: 1.00 AND 3.00.PROBABILITY DIFFERENCE: 0.0263.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.140399130	0.019419963	-0.010348277
2	0.359600870	0.208749001	0.079235497
3		0.151589955	0.103520677
4		0.120241080	0.090805188
5			0.070359149
6			0.054038421
7			0.053315920
8			0.059073425

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.238211067	0.114059302	0.055258195

DISTRIBUTION (10, 1)

CUTPOINTS: 0.25 AND 2.50.PROBABILITY DIFFERENCE: 0.3556.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.024754219	-0.005913480	-0.002366707
2	0.524754219	-0.013729141	-0.005623462
3		0.006042194	-0.007758309
4		0.513600427	-0.004427849
5			0.049426470
6			0.182120814
7			0.159941032
8			0.128688010

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.045184980	0.004121334	0.001544010

DISTRIBUTION (10, 2)

CUTPOINTS: 0.25 AND 2.50.PROBABILITY DIFFERENCE: 0.0193.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.204751671	0.092883489	0.041309863
2	0.295248329	0.100928634	0.056564578
3		0.117730166	0.056962173
4		0.188457710	0.034642051
5			0.024565511
6			0.042593724
7			0.094536528
8			0.148825572

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.248374889	0.123226888	0.060732477

DISTRIBUTION (11, 1)

CUTPOINTS: 0.75 AND 3.00.PROBABILITY DIFFERENCE: 0.2153.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	-0.016674910	-0.013933067	-0.007981888
2	0.516674910	0.051168701	-0.008297763
3		0.310299324	0.089912677
4		0.152465042	0.204330038
5			0.059864840
6			0.038686495
7			0.054596976
8			0.068888625

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.070953959	0.029166191	0.012993799

DISTRIBUTION (11, 2)

CUTPOINTS: 0.75 AND 3.00.PROBABILITY DIFFERENCE: 0.0193.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.160267379	0.040085860	0.001358894
2	0.339732621	0.157048009	0.071003938
3		0.145924782	0.069599345
4		0.156941349	0.064231181
5			0.063851168
6			0.067716892
7			0.076685204
8			0.085553378

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.242821845	0.118803951	0.058528262

DISTRIBUTION (11, 3)

CUTPOINTS: 0.75 AND 3.00.PROBABILITY DIFFERENCE: 0.0151.

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16
1	0.178099015	0.052304945	0.006102534
2	0.321900985	0.156104455	0.075270977
3		0.140638755	0.072916044
4		0.150951845	0.063003383
5			0.059933401
6			0.063858639
7			0.074389623
8			0.084525401

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16
0.245560458	0.120945253	0.059740695

APPENDIX 3

THE EXPECTED VALUES OF THE ORDER STATISTICS FOR SAMPLES OF SIZE 4, 8 AND 16 FROM THE NEAR-NORMAL DISTRIBUTIONS

DISTRIBUTION (1, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.985325146	-1.444422373	-1.910706975
2	-0.241650124	-0.725131694	-1.229176934
3		-0.358430654	-0.836713293
4		-0.110345577	-0.585367201
5			-0.413589305
6			-0.280187789
7			-0.163515603
8			-0.053844534

DISTRIBUTION (1, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.025251688	-1.433838377	-1.799129642
2	-0.287326002	-0.831892056	-1.284374965
3		-0.451285682	-0.966191757
4		-0.144192344	-0.730971072
5			-0.539673487
6			-0.372703892
7			-0.219125642
8			-0.072342364

DISTRIBUTION (2, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.991087283	-1.399911225	-1.814751014
2	-0.272757702	-0.779256143	-1.169390116
3		-0.436798228	-0.898398636
4		-0.142037263	-0.702171511
5			-0.530048699
6			-0.370761369
7			-0.219442013
8			-0.072651926

DISTRIBUTION (2, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.012696749	-1.416509618	-1.798532809
2	-0.284881008	-0.816337503	-1.228949515
3		-0.454352687	-0.945728085
4		-0.146886285	-0.733068454
5			-0.549209696
6			-0.381979699
7			-0.225224264
8			-0.074425068

DISTRIBUTION (3, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.844404577	-1.372757279	-2.008719063
2	-0.137186465	-0.469756348	-1.039007424
3		-0.155334003	-0.527792665
4		-0.037782882	-0.265935548
5			-0.143600574
6			-0.083124187
7			-0.045211488
8			-0.014522803

DISTRIBUTION (3, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.022372160	-1.432082399	-1.802258830
2	-0.285310662	-0.827083279	-1.274257074
3		-0.447501233	-0.963539282
4		-0.142163112	-0.729807669
5			-0.535384298
6			-0.365527338
7			-0.212394203
8			-0.069607936

DISTRIBUTION (3, 3)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.024764390	-1.430807422	-1.793432904
2	-0.288393045	-0.834155305	-1.278998697
3		-0.453819851	-0.971332347
4		-0.144647637	-0.738500024
5			-0.543867146
6			-0.372754347
7			-0.217299705
8			-0.071350753

DISTRIBUTION (4, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.819176407	-1.285071410	-1.908925380
2	-0.162549108	-0.485626189	-0.849375984
3		-0.242121207	-0.529703603
4		-0.077037765	-0.388167370
5			-0.287878028
6			-0.199756644
7			-0.117600089
8			-0.038822433

DISTRIBUTION (4, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.840466184	-1.303503925	-1.907614893
2	-0.173908266	-0.517705807	-0.890036943
3		-0.260753433	-0.571722873
4		-0.082781674	-0.419434158
5			-0.309698513
6			-0.214155118
7			-0.125806581
8			-0.041488746

DISTRIBUTION (4, 3)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.931068712	-1.383436773	-1.907222189
2	-0.221640945	-0.651880227	-1.063651684
3		-0.339434748	-0.741753879
4		-0.107523582	-0.550480258
5			-0.403485836
6			-0.276872132
7			-0.161838783
8			-0.053244027

DISTRIBUTION (4, 4)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.981109696	-1.416009688	-1.871342054
2	-0.253858385	-0.739641322	-1.169428127
3		-0.394711864	-0.851229697
4		-0.125798187	-0.638975166
5			-0.470892659
6			-0.324952284
7			-0.190914234
8			-0.062997106

DISTRIBUTION (4, 5)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.985820214	-1.422091563	-1.876485683
2	-0.255356960	-0.744395432	-1.179082788
3		-0.396709511	-0.856218086
4		-0.126402172	-0.642158907
5			-0.473129421
6			-0.326492737
7			-0.191829907
8			-0.063302086

DISTRIBUTION (4, 6)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.995687889	-1.422293992	-1.851638993
2	-0.264863334	-0.768981803	-1.201146463
3		-0.414149306	-0.888099105
4		-0.132244453	-0.670712103
5			-0.495172326
6			-0.341769992
7			-0.200753964
8			-0.066233868

DISTRIBUTION (4, 7)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.012302075	-1.427394546	-1.822214349
2	-0.278419961	-0.804973173	-1.240450351
3		-0.438189142	-0.932660386
4		-0.140327094	-0.709373952
5			-0.525379064
6			-0.363107381
7			-0.213404034
8			-0.070421660

DISTRIBUTION (4, 8)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.035122280	-1.417794029	-1.734191256
2	-0.305620415	-0.873212031	-1.301526682
3		-0.489561475	-1.015065484
4		-0.158637747	-0.788400848
5			-0.592211872
6			-0.413248227
7			-0.244364367
8			-0.080884143

DISTRIBUTION (5, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.856020209	-1.230099305	-1.656756607
2	-0.226333842	-0.642508480	-0.942473854
3		-0.365959407	-0.744478547
4		-0.119391771	-0.588894108
5			-0.445939887
6			-0.311876567
7			-0.184373447
8			-0.060986829

DISTRIBUTION (5, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.006620360	-1.409614227	-1.794040670
2	-0.282478535	-0.809057389	-1.218272931
3		-0.450784471	-0.936502467
4		-0.145978261	-0.725812113
5			-0.545003308
6			-0.380213549
7			-0.224773355
8			-0.074387047

DISTRIBUTION (6, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.750136520	-1.270280776	-1.982767715
2	-0.099357488	-0.344113273	-0.816756946
3		-0.108320504	-0.351965591
4		-0.028086248	-0.174735912
5			-0.104419424
6			-0.066735741
7			-0.038208693
8			-0.012499473

DISTRIBUTION (6, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.889409314	-1.395330197	-1.997495537
2	-0.171604987	-0.547714657	-1.054763840
3		-0.227794934	-0.625312044
4		-0.063284202	-0.390066817
5			-0.245774863
6			-0.149407020
7			-0.080186482
8			-0.025243154

DISTRIBUTION (6, 3)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.972906912	-1.435492422	-1.921815242
2	-0.233904781	-0.704940105	-1.186381183
3		-0.345132157	-0.820958584
4		-0.103708453	-0.580118870
5			-0.399334269
6			-0.256808359
7			-0.142031189
8			-0.045236789

DISTRIBUTION (6, 4)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.990236468	-1.439213691	-1.893218092
2	-0.249290604	-0.742508860	-1.215977770
3		-0.375016692	-0.865829571
4		-0.114576496	-0.625556711
5			-0.438811050
6			-0.286888948
7			-0.160731720
8			-0.051564866

DISTRIBUTION (6, 5)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.014330532	-1.437636328	-1.834162186
2	-0.274209196	-0.802235065	-1.260432084
3		-0.424242227	-0.936178550
4		-0.132914060	-0.698137763
5			-0.503922734
6			-0.338476073
7			-0.193957813
8			-0.063049090

DISTRIBUTION (6, 6)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.024454049	-1.432492345	-1.798773721
2	-0.287198281	-0.831557840	-1.278154299
3		-0.451255407	-0.968033195
4		-0.143613493	-0.735224495
5			-0.540619854
6			-0.369678290
7			-0.214997355
8			-0.070488754

DISTRIBUTION (7, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.713942201	-1.179144942	-1.854270702
2	-0.112642828	-0.349777187	-0.681999664
3		-0.156466914	-0.365298814
4		-0.048556077	-0.250243462
5			-0.181529715
6			-0.124883906
7			-0.073203016
8			-0.024117783

DISTRIBUTION (7, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.810178821	-1.300969483	-1.962901817
2	-0.145198412	-0.446757125	-0.861341371
3		-0.204870830	-0.465306884
4		-0.064911580	-0.321022688
5			-0.239620298
6			-0.169885610
7			-0.101816256
8			-0.033933595

DISTRIBUTION (7, 3)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.880681073	-1.367221779	-1.967297584
2	-0.179381776	-0.550129298	-0.997005924
3		-0.255973780	-0.616571096
4		-0.077529695	-0.420007012
5			-0.292327375
6			-0.195041486
7			-0.112442642
8			-0.036786983

DISTRIBUTION (7, 4)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.893551494	-1.385131186	-1.984041549
2	-0.182760657	-0.561865133	-1.025000005
3		-0.259630382	-0.628502502
4		-0.078468874	-0.425966403
5			-0.295858947
6			-0.197241451
7			-0.113681463
8			-0.037190075

DISTRIBUTION (7, 5)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.954665710	-1.418272531	-1.930778447
2	-0.225813994	-0.675281643	-1.133105745
3		-0.337020304	-0.776789416
4		-0.103741728	-0.555612308
5			-0.392645350
6			-0.262009483
7			-0.150445070
8			-0.049076641

DISTRIBUTION (7, 6)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.004790423	-1.432098659	-1.849893869
2	-0.268260119	-0.782482226	-1.229201660
3		-0.416697809	-0.907883282
4		-0.131751302	-0.680144219
5			-0.496195749
6			-0.337987615
7			-0.196308348
8			-0.064350694

DISTRIBUTION (7, 7)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.018288737	-1.430324292	-1.810642414
2	-0.282432719	-0.817998336	-1.260564437
3		-0.443494804	-0.951267181
4		-0.141315542	-0.721163417
5			-0.530726349
6			-0.364213919
7			-0.212813063
8			-0.070001764

DISTRIBUTION (7, 8)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.034584997	-1.418718101	-1.738283006
2	-0.304649565	-0.870696972	-1.300266047
3		-0.487747735	-1.011469043
4		-0.158133154	-0.784641114
5			-0.589690944
6			-0.412205120
7			-0.244220331
8			-0.080934693

DISTRIBUTION (7, 9)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.037067275	-1.413091895	-1.717593238
2	-0.310121803	-0.882692957	-1.305584117
3		-0.499427148	-1.024541862
4		-0.162787436	-0.800766029
5			-0.605739557
6			-0.425770916
7			-0.253317239
8			-0.084142527

DISTRIBUTION (8, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.729907958	-1.148446527	-1.738772730
2	-0.144459929	-0.422823997	-0.696134675
3		-0.222612452	-0.468332801
4		-0.071774716	-0.357969273
5			-0.268413896
6			-0.186729614
7			-0.110023718
8			-0.036333563

DISTRIBUTION (8, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.794402525	-1.230874317	-1.818195919
2	-0.165924916	-0.486633860	-0.803176753
3		-0.254912134	-0.541223328
4		-0.081925701	-0.409820541
5			-0.306319441
6			-0.212927456
7			-0.125427372
8			-0.041416267

DISTRIBUTION (8, 3)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.871313140	-1.312011974	-1.853852086
2	-0.199572548	-0.585563253	-0.951337223
3		-0.306768327	-0.664831543
4		-0.097517447	-0.497068394
5			-0.365804912
6			-0.251532954
7			-0.147188206
8			-0.048447857

DISTRIBUTION (8, 4)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.921398447	-1.360284752	-1.864757419
2	-0.223889491	-0.654957748	-1.048434840
3		-0.345928368	-0.750425275
4		-0.109828774	-0.561683415
5			-0.412308422
6			-0.282885010
7			-0.165299259
8			-0.054371894

DISTRIBUTION (8, 5)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.980589548	-1.407912098	-1.849117128
2	-0.257311563	-0.748386817	-1.168344175
3		-0.401442006	-0.866934709
4		-0.127396182	-0.654587123
5			-0.479797604
6			-0.327428256
7			-0.190242054
8			-0.062356379

DISTRIBUTION (8, 6)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.013423324	-1.421616240	-1.803991183
2	-0.282522290	-0.814213290	-1.242109980
3		-0.446847197	-0.944994595
4		-0.143442167	-0.723205878
5			-0.537142328
6			-0.371543533
7			-0.218336613
8			-0.072030527

DISTRIBUTION (9, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.807654242	-1.216298500	-1.747495709
2	-0.186510100	-0.535782557	-0.825517766
3		-0.296085602	-0.608235447
4		-0.096280775	-0.476221349
5			-0.359741322
6			-0.251249502
7			-0.148401570
8			-0.049066435

DISTRIBUTION (9, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.894476571	-1.312292366	-1.804617426
2	-0.222620852	-0.640817639	-0.986388557
3		-0.352470728	-0.733367235
4		-0.114022354	-0.567845641
5			-0.426346430
6			-0.296788972
7			-0.174961463
8			-0.057794045

DISTRIBUTION (9, 3)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.968684912	-1.385900475	-1.825289570
2	-0.257588360	-0.741240251	-1.134338954
3		-0.408132597	-0.854023184
4		-0.131765416	-0.657668123
5			-0.492427325
6			-0.342720997
7			-0.202148562
8			-0.066801914

DISTRIBUTION (9, 4)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.992531768	-1.395351323	-1.787589123
2	-0.275882674	-0.790770940	-1.193269339
3		-0.439758843	-0.915348827
4		-0.142306735	-0.708064339
5			-0.531360704
6			-0.370589403
7			-0.218967404
8			-0.072430986

DISTRIBUTION (10, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.642842133	-1.121246156	-1.848966797
2	-0.071647462	-0.243508133	-0.582534353
3		-0.081528419	-0.229993170
4		-0.023295679	-0.128128332
5			-0.086672356
6			-0.058536087
7			-0.034115157
8			-0.011216452

DISTRIBUTION (10, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.023919773	-1.430772038	-1.795206641
2	-0.287513661	-0.832371431	-1.276556071
3		-0.451896714	-0.970674754
4		-0.143655679	-0.737274243
5			-0.541216593
6			-0.369384516
7			-0.214509569
8			-0.070273148

DISTRIBUTION (11, 1)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-0.686826684	-1.079053131	-1.640515531
2	-0.137162932	-0.397946651	-0.635616065
3		-0.214389007	-0.445476370
4		-0.069395066	-0.345218763
5			-0.259540598
6			-0.180688217
7			-0.106502317
8			-0.035176688

DISTRIBUTION (11, 2)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.007270511	-1.415338324	-1.800978119
2	-0.279665326	-0.806290637	-1.231332346
3		-0.442070031	-0.935826462
4		-0.141860466	-0.715475107
5			-0.531158945
6			-0.367399156
7			-0.215939686
8			-0.071250414

DISTRIBUTION (11, 3)

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

I	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16
1	-1.013098349	-1.418954120	-1.796833299
2	-0.283543785	-0.816564919	-1.242844310
3		-0.448958487	-0.948172005
4		-0.144202316	-0.726498841
5			-0.539932553
6			-0.373619820
7			-0.219610881
8			-0.072459871

APPENDIX 4

THE COVARIANCES OF THE ORDER STATISTICS FOR SAMPLES OF SIZE 4, 8 AND 16 FROM THE NEAR-NORMAL DISTRIBUTIONS

Note that the covariances not given can be found from considerations of symmetry.

COVARIANCE[X[R],X[S]] FOR SAMPLE SIZE 4

[R, S]	DISTRIBUTION (1,1)	DISTRIBUTION (1,2)	DISTRIBUTION (2,1)	DISTRIBUTION (2,2)
[1, 1]	0.705017357	0.524444045	0.638183497	0.561225298
[1, 2]	0.238019599	0.246019997	0.230951999	0.239530390
[1, 3]	0.137924293	0.155904725	0.145822890	0.152619312
[1, 4]	0.118798207	0.108614836	0.104200425	0.105376542
[2, 2]	0.265722218	0.341858700	0.305165737	0.332062809
[2, 3]	0.158574433	0.221232975	0.198900564	0.217035947

[R, S]	DISTRIBUTION (3,1)	DISTRIBUTION (3,2)	DISTRIBUTION (3,3)	DISTRIBUTION (4,1)
[1, 1]	1.099280212	0.535971758	0.523189309	1.165502106
[1, 2]	0.200971369	0.244978900	0.245435905	0.175752556
[1, 3]	0.076469484	0.154912501	0.155898148	0.087406032
[1, 4]	0.118765989	0.108680855	0.107837173	0.098982448
[2, 2]	0.168880572	0.337381235	0.343498088	0.137025696
[2, 3]	0.058191521	0.218183351	0.222807323	0.072172574

[R, S]	DISTRIBUTION (4,2)	DISTRIBUTION (4,3)	DISTRIBUTION (4,4)	DISTRIBUTION (4,5)
[1, 1]	1.115247342	0.868209736	0.700754684	0.687110746
[1, 2]	0.182541846	0.214009908	0.231158986	0.233319256
[1, 3]	0.094142194	0.123059146	0.140453084	0.141808514
[1, 4]	0.100993503	0.109447036	0.110742258	0.111658416
[2, 2]	0.148125167	0.215776609	0.272225001	0.275840583
[2, 3]	0.082265908	0.132428511	0.173053918	0.175134715

[R, S]	DISTRIBUTION (4,6)	DISTRIBUTION (4,7)	DISTRIBUTION (4,8)	DISTRIBUTION (5,1)
[1, 1]	0.645115359	0.576837561	0.455503209	1.005309004
[1, 2]	0.235911981	0.241209685	0.246545821	0.174414339
[1, 3]	0.145707260	0.151684503	0.160123362	0.108145017
[1, 4]	0.110141937	0.108806514	0.101908630	0.082050561
[2, 2]	0.293337684	0.320889273	0.379614817	0.210693389
[2, 3]	0.188166539	0.207678276	0.249634978	0.136828334

[R, S]	DISTRIBUTION (5,2)	DISTRIBUTION (6,1)	DISTRIBUTION (6,2)	DISTRIBUTION (6,3)
[1, 1]	0.580677180	1.309177938	1.006999104	0.749341626
[1, 2]	0.236835570	0.166580181	0.210391726	0.233575266
[1, 3]	0.150767461	0.052425056	0.098643728	0.133535860
[1, 4]	0.104497305	0.102587700	0.118994672	0.118293889
[2, 2]	0.326244148	0.118245353	0.172503697	0.249399069
[2, 3]	0.213375307	0.031978534	0.083431617	0.148743165

[R, S]	DISTRIBUTION (6,4)	DISTRIBUTION (6,5)	DISTRIBUTION (6,6)	DISTRIBUTION (7,1)
[1, 1]	0.683765796	0.579059907	0.526688320	1.387883925
[1, 2]	0.238113893	0.243774356	0.245604418	0.142977488
[1, 3]	0.140948905	0.151297620	0.155683588	0.057177257
[1, 4]	0.116534925	0.111881337	0.108392233	0.086160765
[2, 2]	0.273520136	0.316882982	0.341322729	0.089714201
[2, 3]	0.168053547	0.202031824	0.221020706	0.035931619

[R, S]	DISTRIBUTION (7,2)	DISTRIBUTION (7,3)	DISTRIBUTION (7,4)	DISTRIBUTION (7,5)
[1, 1]	1.182771214	1.026260117	0.993641265	0.807312045
[1, 2]	0.177901004	0.201971920	0.207751266	0.225837541
[1, 3]	0.079428827	0.101447941	0.104453600	0.128325773
[1, 4]	0.103522210	0.112229235	0.115172084	0.115808886
[2, 2]	0.139756486	0.165962909	0.174523005	0.230309378
[2, 3]	0.059290430	0.088708016	0.092253916	0.138243064

[R, S]	DISTRIBUTION (7,6)	DISTRIBUTION (7,7)	DISTRIBUTION (7,8)	DISTRIBUTION (7,9)
[1, 1]	0.615985437	0.552921648	0.459384022	0.438748941
[1, 2]	0.240072456	0.243466848	0.246493619	0.246476334
[1, 3]	0.148282292	0.153598594	0.159928693	0.160884774
[1, 4]	0.111631392	0.108781606	0.102264315	0.100053919
[2, 2]	0.302447278	0.330398158	0.377438505	0.389566993
[2, 3]	0.193226398	0.213767704	0.248068533	0.256907932

[R, S]	DISTRIBUTION (8,1)	DISTRIBUTION (8,2)	DISTRIBUTION (8,3)	DISTRIBUTION (8,4)
[1, 1]	1.342776961	1.211259389	1.024817811	0.885117021
[1, 2]	0.139363207	0.162376881	0.189682998	0.208080517
[1, 3]	0.069865636	0.084941437	0.106397953	0.121601783
[1, 4]	0.078731125	0.089569000	0.099532455	0.104920851
[2, 2]	0.103588741	0.130134162	0.176166400	0.215781377
[2, 3]	0.056445487	0.074400813	0.107321433	0.134816151

[R, S]	DISTRIBUTION (8,5)	DISTRIBUTION (8,6)	DISTRIBUTION (9,1)	DISTRIBUTION (9,2)
[1, 1]	0.694552993	0.564666536	1.158890824	0.941567447
[1, 2]	0.229826032	0.240654532	0.162330181	0.194350919
[1, 3]	0.140873754	0.152455293	0.092149668	0.115929850
[1, 4]	0.108713014	0.106947799	0.084859672	0.096327512
[2, 2]	0.277681905	0.328487786	0.154017783	0.208784174
[2, 3]	0.177652517	0.213678230	0.093272024	0.132759330

[R, S]	DISTRIBUTION (9,3)	DISTRIBUTION (9,4)	DISTRIBUTION (10,1)	DISTRIBUTION (10,2)
[1, 1]	0.720607532	0.627211458	1.504736589	0.527113977
[1, 2]	0.223142797	0.230946049	0.126476454	0.245219386
[1, 3]	0.138245584	0.146378829	0.034380375	0.155563218
[1, 4]	0.104297322	0.103026064	0.079854719	0.108018789
[2, 2]	0.274690245	0.311557982	0.076884044	0.341810216
[2, 3]	0.177628138	0.203554738	0.016810991	0.221491809

[R, S]	DISTRIBUTION (11,1)	DISTRIBUTION (11,2)	DISTRIBUTION (11,3)
[1, 1]	1.416850475	0.585148642	0.562825951
[1, 2]	0.122972307	0.238048954	0.240191258
[1, 3]	0.062317782	0.150515910	0.152464567
[1, 4]	0.069261337	0.106281428	0.106263439
[2, 2]	0.092604962	0.322044781	0.330408707
[2, 3]	0.050703050	0.209395420	0.215190252

COVARIANCE[X[R],X[S]] FOR SAMPLE SIZE 8

[R, S]	DISTRIBUTION (1,1)	DISTRIBUTION (1,2)	DISTRIBUTION (2,1)	DISTRIBUTION (2,2)
[1, 1]	0.697831591	0.414961611	0.689264119	0.520347293
[1, 2]	0.276684773	0.204124657	0.203977246	0.197178217
[1, 3]	0.141068100	0.130484677	0.129732059	0.129658868
[1, 4]	0.090701597	0.095212581	0.098775381	0.097577458
[1, 5]	0.069776385	0.074817639	0.077828495	0.076806327
[1, 6]	0.061415659	0.061427932	0.061399276	0.061423986
[1, 7]	0.062007214	0.051692784	0.048234116	0.048908740
[1, 8]	0.060621047	0.041208808	0.044938196	0.041417761
[2, 2]	0.301477710	0.254285836	0.204343674	0.221486771
[2, 3]	0.149883588	0.162855902	0.133783806	0.148854663
[2, 4]	0.094325055	0.119018031	0.103009779	0.113078423
[2, 5]	0.071922466	0.093645158	0.081737105	0.089548485
[2, 6]	0.063035633	0.076972342	0.064799294	0.071904907
[2, 7]	0.063452088	0.064826645	0.051193146	0.057448003
[3, 3]	0.146882922	0.190416895	0.166487536	0.183721859
[3, 4]	0.092334630	0.139982181	0.129207258	0.140418113
[3, 5]	0.070639890	0.110592494	0.102904151	0.111590459
[3, 6]	0.062368188	0.091189808	0.081782146	0.089812197
[4, 4]	0.100987132	0.167948576	0.161945822	0.173525717
[4, 5]	0.078789337	0.133509062	0.129269427	0.138343106

[R, S]	DISTRIBUTION (3,1)	DISTRIBUTION (3,2)	DISTRIBUTION (3,3)	DISTRIBUTION (4,1)
[1, 1]	1.340595320	0.439480115	0.418219404	1.720240115
[1, 2]	0.414942168	0.205229375	0.200663490	0.306263028
[1, 3]	0.126058488	0.131694464	0.130363809	0.111826718
[1, 4]	0.045299533	0.094865150	0.094813207	0.074081161
[1, 5]	0.029853840	0.074194059	0.074322316	0.057182996
[1, 6]	0.039304884	0.061651032	0.061352053	0.046657328
[1, 7]	0.070598481	0.051850542	0.051073825	0.046981436
[1, 8]	0.098601538	0.041694595	0.040442087	0.078088502
[2, 2]	0.413793102	0.250738118	0.249125809	0.217932693
[2, 3]	0.122205334	0.161970072	0.162880315	0.069429410
[2, 4]	0.038958041	0.117153909	0.118934250	0.043865337
[2, 5]	0.022408779	0.091925244	0.093515892	0.033788938
[2, 6]	0.027534440	0.076539780	0.077344960	0.027680616
[2, 7]	0.049358557	0.064434643	0.064452263	0.028171822
[3, 3]	0.094321980	0.190319649	0.193288295	0.061918282
[3, 4]	0.027280935	0.138278568	0.141783046	0.040774521
[3, 5]	0.013524736	0.108860978	0.111855947	0.031922059
[3, 6]	0.015665725	0.090820605	0.092705520	0.026527513
[4, 4]	0.020599826	0.164067666	0.169466145	0.048110090
[4, 5]	0.011124632	0.130018261	0.134494259	0.038103887

[R, S]	DISTRIBUTION (4,2)	DISTRIBUTION (4,3)	DISTRIBUTION (4,4)	DISTRIBUTION (4,5)
[1, 1]	1.622922013	1.107595853	0.770085461	0.736014407
[1, 2]	0.297893430	0.268989730	0.239544492	0.241117383
[1, 3]	0.117411049	0.134309537	0.136574482	0.137352010
[1, 4]	0.078018157	0.092093613	0.096933844	0.097468926
[1, 5]	0.060086521	0.071025785	0.075460068	0.075900681
[1, 6]	0.049274475	0.058769463	0.061933807	0.062339834
[1, 7]	0.048517241	0.053843921	0.053978674	0.054509815
[1, 8]	0.075196628	0.064040450	0.053774300	0.053939917
[2, 2]	0.209483543	0.219672757	0.230206863	0.235223500
[2, 3]	0.076054473	0.109670892	0.133416489	0.135537247
[2, 4]	0.049353540	0.075699658	0.095598974	0.096943050
[2, 5]	0.038051403	0.058773133	0.074857381	0.075881642
[2, 6]	0.031351211	0.048895298	0.061707338	0.062573702
[2, 7]	0.031170771	0.045062262	0.053997709	0.054922311
[3, 3]	0.070062620	0.113344890	0.147993592	0.149755817
[3, 4]	0.047282626	0.079934995	0.107387563	0.108510688
[3, 5]	0.037045980	0.062769054	0.084695363	0.085564813
[3, 6]	0.030922033	0.052634371	0.070183007	0.070939127
[4, 4]	0.055544882	0.093764095	0.127938722	0.129181357
[4, 5]	0.044017298	0.074335164	0.101643399	0.102623978

[R, S]	DISTRIBUTION (4,6)	DISTRIBUTION (4,7)	DISTRIBUTION (4,8)	DISTRIBUTION (5,1)
[1, 1]	0.661291652	0.528401297	0.310552138	1.559264571
[1, 2]	0.227381660	0.211773972	0.173350948	0.173368030
[1, 3]	0.135619255	0.133037204	0.121365050	0.104535377
[1, 4]	0.097461356	0.097143684	0.092788746	0.080442116
[1, 5]	0.076075675	0.076135358	0.073493914	0.063248100
[1, 6]	0.062289031	0.062086699	0.058669821	0.049459800
[1, 7]	0.053218468	0.051846903	0.046034672	0.037268779
[1, 8]	0.049774895	0.044643082	0.033051007	0.041104833
[2, 2]	0.233096585	0.237832623	0.238325770	0.137039550
[2, 3]	0.141385255	0.151467599	0.167676878	0.088178670
[2, 4]	0.102539082	0.111413043	0.128560943	0.068906679
[2, 5]	0.080493651	0.087723201	0.102017599	0.054826148
[2, 6]	0.066170713	0.071764873	0.081551992	0.043208490
[2, 7]	0.056724182	0.060076410	0.064061735	0.032765134
[3, 3]	0.161056668	0.177710789	0.211441037	0.115164835
[3, 4]	0.117971963	0.131685779	0.162734132	0.090457493
[3, 5]	0.093164526	0.104173483	0.129450429	0.072118506
[3, 6]	0.076913349	0.085504196	0.103665579	0.056907643
[4, 4]	0.141293637	0.158916874	0.202205522	0.114388913
[4, 5]	0.112307760	0.126411134	0.161306962	0.091328142

[R, S]	DISTRIBUTION (5,2)	DISTRIBUTION (6,1)	DISTRIBUTION (6,2)	DISTRIBUTION (6,3)
[1, 1]	0.566119626	1.841315351	1.249950959	0.798164723
[1, 2]	0.196582569	0.416752773	0.362086790	0.280652784
[1, 3]	0.128749304	0.097068463	0.139597947	0.145305106
[1, 4]	0.097120861	0.034947952	0.068239370	0.089170766
[1, 5]	0.076514132	0.023719770	0.047785140	0.066697600
[1, 6]	0.060960945	0.027322415	0.050301666	0.061503674
[1, 7]	0.048493584	0.054598585	0.066085794	0.061896385
[1, 8]	0.041549500	0.105330047	0.086520061	0.062823214
[2, 2]	0.216604529	0.352721160	0.315432586	0.278996251
[2, 3]	0.145819657	0.075908983	0.117309951	0.143794610
[2, 4]	0.111170358	0.022079246	0.055343285	0.088035022
[2, 5]	0.088095661	0.013097971	0.037301169	0.065743547
[2, 6]	0.070470747	0.014203938	0.038395819	0.060592863
[2, 7]	0.056270987	0.027796275	0.050274454	0.060982494
[3, 3]	0.179931040	0.051648054	0.093126213	0.142706864
[3, 4]	0.138057759	0.012835679	0.043264567	0.087589451
[3, 5]	0.109788127	0.006714577	0.028853048	0.065389344
[3, 6]	0.088040234	0.007281234	0.029464119	0.060203663
[4, 4]	0.171242385	0.009766071	0.038657118	0.092681467
[4, 5]	0.136594291	0.005641109	0.027445398	0.070699018

[R, S]	DISTRIBUTION (6,4)	DISTRIBUTION (6,5)	DISTRIBUTION (6,6)	DISTRIBUTION (7,1)
[1, 1]	0.684357484	0.506654397	0.421965294	2.192812313
[1, 2]	0.259255933	0.222822889	0.203092123	0.323193365
[1, 3]	0.143158734	0.136391884	0.131117673	0.087223715
[1, 4]	0.092169765	0.094498007	0.094984666	0.050122823
[1, 5]	0.069934071	0.073204239	0.074375942	0.037879913
[1, 6]	0.062591119	0.062530393	0.061583979	0.032166097
[1, 7]	0.059653951	0.054830771	0.051511861	0.039378179
[1, 8]	0.056687625	0.046374865	0.041075712	0.084787336
[2, 2]	0.271756485	0.259666021	0.250820186	0.213794560
[2, 3]	0.150088076	0.159729598	0.162742530	0.047090965
[2, 4]	0.096694758	0.111032266	0.118321283	0.023828823
[2, 5]	0.073473152	0.086285397	0.092934561	0.017645792
[2, 6]	0.065837313	0.073852138	0.077099939	0.014955070
[2, 7]	0.062771891	0.064803694	0.064553417	0.018342846
[3, 3]	0.156650441	0.180530210	0.192166856	0.034295023
[3, 4]	0.101267900	0.125980680	0.140374295	0.017808958
[3, 5]	0.077024312	0.098147847	0.110631579	0.013469190
[3, 6]	0.069024599	0.084109326	0.091971775	0.011711824
[4, 4]	0.110814843	0.145122447	0.167268626	0.019531641
[4, 5]	0.085638462	0.114126824	0.132637272	0.015198677

[R, S]	DISTRIBUTION (7,2)	DISTRIBUTION (7,3)	DISTRIBUTION (7,4)	DISTRIBUTION (7,5)
[1, 1]	1.662177456	1.361302433	1.271658403	0.935044109
[1, 2]	0.353206424	0.330123601	0.337243515	0.283704535
[1, 3]	0.105102760	0.131621669	0.134039374	0.141698158
[1, 4]	0.065966803	0.077244861	0.078480916	0.090690045
[1, 5]	0.051618310	0.057596894	0.058583063	0.068762523
[1, 6]	0.042378712	0.051628910	0.052769635	0.060266734
[1, 7]	0.050475128	0.058045480	0.060102809	0.059354793
[1, 8]	0.089349648	0.080761619	0.081868658	0.065582246
[2, 2]	0.307311917	0.262095787	0.282178908	0.258524444
[2, 3]	0.073303410	0.098679625	0.104441848	0.127601769
[2, 4]	0.038481629	0.056379081	0.058957944	0.081670420
[2, 5]	0.029335502	0.041636290	0.043406735	0.062057687
[2, 6]	0.024083107	0.037196882	0.038886526	0.054479148
[2, 7]	0.028666168	0.041760750	0.044172867	0.053704696
[3, 3]	0.057375185	0.082706154	0.086554753	0.125362873
[3, 4]	0.030132749	0.048176755	0.049749620	0.081235182
[3, 5]	0.022916051	0.036205036	0.037273757	0.062344917
[3, 6]	0.019192273	0.032808687	0.033887802	0.055079327
[4, 4]	0.034836346	0.050424560	0.051761807	0.089221273
[4, 5]	0.027089835	0.039069846	0.040045322	0.069749215

[R, S]	DISTRIBUTION (7,6)	DISTRIBUTION (7,7)	DISTRIBUTION (7,8)	DISTRIBUTION (7,9)
[1, 1]	0.588674180	0.476222293	0.317005411	0.285475860
[1, 2]	0.228067796	0.208009009	0.174989295	0.166062714
[1, 3]	0.136901453	0.132374074	0.121823929	0.118564609
[1, 4]	0.096372429	0.095794333	0.093173908	0.092170981
[1, 5]	0.074871341	0.074967188	0.073817646	0.073223720
[1, 6]	0.062626331	0.061845065	0.058849764	0.057722758
[1, 7]	0.054482048	0.051907156	0.046314626	0.044509184
[1, 8]	0.048925628	0.042918856	0.033529034	0.031185760
[2, 2]	0.247164541	0.245406347	0.238656649	0.234419843
[2, 3]	0.149796253	0.157724115	0.166952393	0.168047874
[2, 4]	0.106124704	0.114787586	0.128037212	0.130923105
[2, 5]	0.082852896	0.090191430	0.101611074	0.104142974
[2, 6]	0.069520055	0.074595425	0.081118748	0.082196134
[2, 7]	0.060585518	0.062700862	0.063919378	0.063464086
[3, 3]	0.168798732	0.185100659	0.209695718	0.215649681
[3, 4]	0.120479589	0.135460225	0.161484978	0.168643150
[3, 5]	0.094561641	0.106860334	0.128476719	0.134413858
[3, 6]	0.079609334	0.088608213	0.102767048	0.106266226
[4, 4]	0.141182073	0.161664120	0.200864009	0.212551830
[4, 5]	0.111746922	0.128346770	0.160262169	0.169744593

[R, S]	DISTRIBUTION (8,1)	DISTRIBUTION (8,2)	DISTRIBUTION (8,3)	DISTRIBUTION (8,4)
[1, 1]	2.211289658	1.894524459	1.480635395	1.175303681
[1, 2]	0.246571898	0.263351027	0.258670965	0.250268455
[1, 3]	0.089881567	0.105149486	0.122332224	0.130447035
[1, 4]	0.063001716	0.073141705	0.084026134	0.090512333
[1, 5]	0.048848194	0.056684672	0.064695117	0.069852820
[1, 6]	0.038865288	0.045418252	0.053049926	0.057564117
[1, 7]	0.034954605	0.041308644	0.047963884	0.051106493
[1, 8]	0.064643509	0.067661652	0.063700205	0.059425861
[2, 2]	0.147791515	0.165992485	0.183370870	0.200385543
[2, 3]	0.046801026	0.061849601	0.087633568	0.107173883
[2, 4]	0.032229505	0.042722633	0.061010725	0.075509526
[2, 5]	0.025198492	0.033343830	0.047463994	0.058866089
[2, 6]	0.020211388	0.026912916	0.039223450	0.048850793
[2, 7]	0.018508889	0.024847249	0.035772932	0.043641438
[3, 3]	0.047010490	0.061835189	0.091124298	0.115487920
[3, 4]	0.033870665	0.044542528	0.065051292	0.082767479
[3, 5]	0.026825980	0.035225242	0.051202674	0.065123313
[3, 6]	0.021710042	0.028696151	0.042645336	0.054376443
[4, 4]	0.041490866	0.054091752	0.076993234	0.097749802
[4, 5]	0.033034385	0.043049993	0.061109824	0.077544640

[R, S]	DISTRIBUTION (8,5)	DISTRIBUTION (8,6)	DISTRIBUTION (9,1)	DISTRIBUTION (9,2)
[1, 1]	0.771185907	0.514987760	1.833427776	1.346612036
[1, 2]	0.229054671	0.203043990	0.221851298	0.223923219
[1, 3]	0.135275298	0.130913738	0.103742492	0.120642073
[1, 4]	0.095764845	0.096498617	0.077414120	0.088902685
[1, 5]	0.074253868	0.075739349	0.060547171	0.069455510
[1, 6]	0.061510153	0.061487524	0.047528243	0.055173774
[1, 7]	0.052689987	0.050413365	0.038492072	0.045116461
[1, 8]	0.050950129	0.042353326	0.056283828	0.054034217
[2, 2]	0.223182092	0.232206768	0.151145693	0.167905178
[2, 3]	0.135103649	0.152380524	0.066166736	0.094458473
[2, 4]	0.096816592	0.113267803	0.049599792	0.070821634
[2, 5]	0.075756143	0.089368664	0.039239852	0.055936328
[2, 6]	0.063119907	0.072804060	0.031058054	0.044775974
[2, 7]	0.054251494	0.059839281	0.025502759	0.036963821
[3, 3]	0.154159228	0.182701636	0.077595283	0.110948314
[3, 4]	0.111406612	0.136628243	0.059390125	0.084489458
[3, 5]	0.087645240	0.108223661	0.047278150	0.067144177
[3, 6]	0.073268259	0.088399464	0.037574977	0.053964818
[4, 4]	0.131788000	0.165919746	0.074449587	0.104539258
[4, 5]	0.104420962	0.132052944	0.059403888	0.083352826

[R, S]	DISTRIBUTION (9,3)	DISTRIBUTION (9,4)	DISTRIBUTION (10,1)	DISTRIBUTION (10,2)
[1, 1]	0.855528092	0.670340250	2.386820864	0.425211442
[1, 2]	0.216488353	0.197279080	0.361011002	0.201599014
[1, 3]	0.130191552	0.127520400	0.066327006	0.130903355
[1, 4]	0.096312601	0.095768766	0.026923521	0.094614112
[1, 5]	0.075475916	0.075365133	0.019280131	0.074036508
[1, 6]	0.060359608	0.060103273	0.018685212	0.061454293
[1, 7]	0.049265212	0.047995610	0.035511509	0.051271160
[1, 8]	0.048853315	0.042290695	0.095411003	0.040697400
[2, 2]	0.201160708	0.209146288	0.259494480	0.249123129
[2, 3]	0.125574029	0.139840975	0.039824695	0.162941203
[2, 4]	0.094306589	0.106337410	0.011743439	0.118291386
[2, 5]	0.074515127	0.084234182	0.007582153	0.092897764
[2, 6]	0.059932616	0.067477322	0.007108202	0.077282601
[2, 7]	0.049207702	0.054108487	0.013140809	0.064540200
[3, 3]	0.149556902	0.171774624	0.024654567	0.193332489
[3, 4]	0.113463239	0.131467382	0.006156761	0.140906222
[3, 5]	0.090104650	0.104514080	0.003767688	0.110969060
[3, 6]	0.072723800	0.083938256	0.003679422	0.092462900
[4, 4]	0.139662720	0.162775797	0.005351363	0.167534523
[4, 5]	0.111327775	0.129818378	0.003604854	0.132764564

[R, S]	DISTRIBUTION (11,1)	DISTRIBUTION (11,2)	DISTRIBUTION (11,3)
[1, 1]	2.418053174	0.561056239	0.514939231
[1, 2]	0.215794197	0.203282616	0.200267149
[1, 3]	0.079516839	0.130371710	0.130016099
[1, 4]	0.057237092	0.095895704	0.096056878
[1, 5]	0.044454798	0.075224660	0.075430265
[1, 6]	0.035111912	0.061095521	0.061174497
[1, 7]	0.030018300	0.050191651	0.049953356
[1, 8]	0.057022743	0.042664054	0.041597245
[2, 2]	0.127461192	0.228705937	0.230876639
[2, 3]	0.039337814	0.149720856	0.152681076
[2, 4]	0.028016809	0.111144706	0.113759009
[2, 5]	0.022026959	0.087675240	0.089800763
[2, 6]	0.017563575	0.071469972	0.073080336
[2, 7]	0.015296491	0.058868054	0.059819463
[3, 3]	0.042255327	0.179104978	0.183970406
[3, 4]	0.031270954	0.133766005	0.137851467
[3, 5]	0.024835056	0.105937923	0.109224798
[3, 6]	0.019934772	0.086592078	0.089112639
[4, 4]	0.038734790	0.162295378	0.167646632
[4, 5]	0.030872898	0.129160149	0.133446357

COVARIANCE[X[R],X[S]] FOR SAMPLE SIZE 16

[R, S]	DISTRIBUTION (1,1)	DISTRIBUTION (1, 2)	DISTRIBUTION (2,1)	DISTRIBUTION (2, 2)
[1, 1]	0.609880303	0.318577790	0.832682251	0.541709887
[1, 2]	0.288365525	0.167461035	0.229949216	0.187398909
[1, 3]	0.171450689	0.110943311	0.121942646	0.113325882
[1, 4]	0.105237059	0.081188975	0.092326197	0.086565941
[1, 5]	0.070999883	0.063656877	0.077001662	0.071128808
[1, 6]	0.054104715	0.052495179	0.066039882	0.060143357
[1, 7]	0.044795591	0.044778256	0.057430459	0.051761122
[1, 8]	0.038665902	0.039031896	0.050414595	0.045180210
[1, 9]	0.034152445	0.034530138	0.044561247	0.039912866
[1,10]	0.030710504	0.030883845	0.039533062	0.035574039
[1,11]	0.028255606	0.027872046	0.035024208	0.031821675
[1,12]	0.027236263	0.025398265	0.030782199	0.028365493
[1,13]	0.028404333	0.023401705	0.026653660	0.024997382
[1,14]	0.031290121	0.021658638	0.022858443	0.021741259
[1,15]	0.032323833	0.019548893	0.021227460	0.019423627
[1,16]	0.027846210	0.015800304	0.025475192	0.019674762
[2, 2]	0.347791016	0.208019459	0.195141694	0.180764532
[2, 3]	0.206145263	0.137702237	0.100573634	0.109725037
[2, 4]	0.125062393	0.100689178	0.075628282	0.084081121
[2, 5]	0.083597523	0.078893182	0.063150458	0.069269237
[2, 6]	0.063416920	0.065029031	0.054263018	0.058697724
[2, 7]	0.052392426	0.055452361	0.047267314	0.050609995
[2, 8]	0.045150227	0.048325682	0.041554297	0.044247035
[2, 9]	0.039821369	0.042744753	0.036776803	0.039142846
[2,10]	0.035760530	0.038225794	0.032660024	0.034925950
[2,11]	0.032853216	0.034494096	0.028956189	0.031266364
[2,12]	0.031601883	0.031429267	0.025462739	0.027885242
[2,13]	0.032886520	0.028955756	0.022058939	0.024584197
[2,14]	0.036212740	0.026797158	0.018938671	0.021395139
[2,15]	0.037469474	0.024186453	0.017642714	0.019143438
[3, 3]	0.214539974	0.152772679	0.103149436	0.119272885
[3, 4]	0.128434903	0.111638824	0.079112179	0.092400545
[3, 5]	0.083933703	0.087434778	0.066488965	0.076453747
[3, 6]	0.062640756	0.072050428	0.057346519	0.064957516
[3, 7]	0.051354045	0.061432353	0.050089996	0.056120690
[3, 8]	0.044095584	0.053535455	0.044133131	0.049147767
[3, 9]	0.038799346	0.047352965	0.039131356	0.043539970
[3,10]	0.034776679	0.042347463	0.034801961	0.038892892
[3,11]	0.031892634	0.038214624	0.030888163	0.034845658
[3,12]	0.030609950	0.034820832	0.027182338	0.031094295
[3,13]	0.031774837	0.032081621	0.023564396	0.027424326
[3,14]	0.034952512	0.029690237	0.020256842	0.023880052
[4, 4]	0.120333908	0.120045297	0.091117257	0.105356281
[4, 5]	0.077754677	0.094114170	0.076880741	0.087382260
[4, 6]	0.057306985	0.077613772	0.066414454	0.074332692
[4, 7]	0.046709920	0.066215234	0.058071825	0.064276311
[4, 8]	0.040043807	0.057732680	0.051207926	0.056329842
[4, 9]	0.035223660	0.051087691	0.045434935	0.049931735
[4,10]	0.031572057	0.045704350	0.040429549	0.044622892
[4,11]	0.028963732	0.041258001	0.035896764	0.039992559
[4,12]	0.027820710	0.037606880	0.031598772	0.035695045
[4,13]	0.028897622	0.034659269	0.027399607	0.031487293
[5, 5]	0.073407793	0.101409319	0.088458996	0.098815410
[5, 6]	0.054293502	0.083777351	0.076466794	0.084111375
[5, 7]	0.044304100	0.071561488	0.066889578	0.072765946
[5, 8]	0.038050230	0.062452698	0.059003471	0.063794814
[5, 9]	0.033541818	0.055307067	0.052366324	0.056567173
[5,10]	0.030126946	0.049510966	0.046607515	0.050565555
[5,11]	0.027702475	0.044719774	0.041388670	0.045326721
[5,12]	0.026695648	0.040784819	0.036437394	0.040461014
[6, 6]	0.055153685	0.091352086	0.086735632	0.094041112
[6, 7]	0.045323792	0.078152371	0.075900426	0.081398021
[6, 8]	0.039062675	0.068280467	0.066972711	0.071394407
[6, 9]	0.034526110	0.060521530	0.059454515	0.063329382
[6,10]	0.031084352	0.054218896	0.052926892	0.056626635
[6,11]	0.028654387	0.049003995	0.047007424	0.050770284
[7, 7]	0.048712733	0.086168043	0.085356960	0.090616956
[7, 8]	0.042132057	0.075371329	0.075347502	0.079531268
[7, 9]	0.037316680	0.066866721	0.066911485	0.070584736
[7,10]	0.033655432	0.059948189	0.059580444	0.063140005
[8, 8]	0.046561718	0.083918458	0.084549632	0.088789951
[8, 9]	0.041315179	0.074522937	0.075118214	0.078861673

[R, S]	DISTRIBUTION (3,1)	DISTRIBUTION (3,2)	DISTRIBUTION (3,3)	DISTRIBUTION (4,1)
[1, 1]	1.275394536	0.367663406	0.340194843	2.311027157
[1, 2]	0.575252687	0.171825441	0.164213320	0.600857818
[1, 3]	0.291846274	0.113166707	0.109196047	0.192839237
[1, 4]	0.131630684	0.084760087	0.082238611	0.098320440
[1, 5]	0.057501403	0.067257659	0.065620648	0.072761960
[1, 6]	0.029142297	0.054847621	0.053893649	0.060518584
[1, 7]	0.019055893	0.045590393	0.045152505	0.051760101
[1, 8]	0.015044092	0.038836428	0.038697567	0.044924807
[1, 9]	0.013126635	0.034186130	0.034109866	0.039560099
[1,10]	0.012436447	0.031068268	0.030873823	0.035266814
[1,11]	0.013555844	0.028764450	0.028378660	0.031615089
[1,12]	0.018544645	0.026660096	0.026110013	0.028315369
[1,13]	0.030217259	0.024436967	0.023795322	0.025880842
[1,14]	0.047777442	0.022065101	0.021387761	0.027693416
[1,15]	0.060785584	0.019603977	0.018890037	0.042083131
[1,16]	0.056708383	0.016701061	0.015832085	0.069345210
[2, 2]	0.690393231	0.199352040	0.194023985	0.505709551
[2, 3]	0.351988600	0.131806131	0.129418351	0.148244578
[2, 4]	0.156561527	0.098917058	0.097631192	0.065933603
[2, 5]	0.066689632	0.078576466	0.077977812	0.045932617
[2, 6]	0.032835332	0.064125666	0.064086013	0.037681252
[2, 7]	0.021029222	0.053342412	0.053726907	0.032087182
[2, 8]	0.016411867	0.045479592	0.046079150	0.027760825
[2, 9]	0.014205735	0.040068387	0.040644226	0.024373195
[2,10]	0.013340934	0.036437004	0.036806816	0.021674122
[2,11]	0.014357978	0.033745645	0.033841263	0.019396487
[2,12]	0.019401383	0.031279997	0.031139378	0.017352239
[2,13]	0.031567574	0.028672910	0.028380764	0.015829913
[2,14]	0.050324789	0.025892304	0.025511829	0.016846751
[2,15]	0.064707728	0.023007843	0.022535679	0.025473054
[3, 3]	0.349572677	0.147111684	0.145352893	0.106979770
[3, 4]	0.157071649	0.110656561	0.109892099	0.044481702
[3, 5]	0.064648058	0.088009888	0.087878142	0.029718747
[3, 6]	0.029612752	0.071880986	0.072281278	0.024216286
[3, 7]	0.017754224	0.059838432	0.060642107	0.020653855
[3, 8]	0.013389335	0.051062689	0.052051501	0.017919273
[3, 9]	0.011392967	0.045027436	0.045948040	0.015775228
[3,10]	0.010552151	0.040974395	0.041634521	0.014057327
[3,11]	0.011155993	0.037961256	0.038292500	0.012596276
[3,12]	0.014798349	0.035191713	0.035240030	0.011280434
[3,13]	0.023962349	0.032260060	0.032120586	0.010321286
[3,14]	0.038571219	0.029134088	0.028876658	0.011075514
[4, 4]	0.130883579	0.122023539	0.121696261	0.038396221
[4, 5]	0.053234445	0.097152967	0.097425261	0.026460957
[4, 6]	0.022545904	0.079395381	0.080187587	0.021741599
[4, 7]	0.012327973	0.066125200	0.067312204	0.018641500
[4, 8]	0.008839540	0.056457560	0.057809931	0.016254799
[4, 9]	0.007367255	0.049813258	0.051060612	0.014377345
[4,10]	0.006741662	0.045350636	0.046288115	0.012860439
[4,11]	0.007038039	0.042026781	0.042583695	0.011553172
[4,12]	0.009208433	0.038964430	0.039193590	0.010365153
[4,13]	0.014820289	0.035719682	0.035726270	0.009519616
[5, 5]	0.039723139	0.105832605	0.106658820	0.028146308
[5, 6]	0.016182373	0.086521315	0.087830807	0.023394000
[5, 7]	0.008155769	0.072075905	0.073754834	0.020120086
[5, 8]	0.005559695	0.061549525	0.063364513	0.017580974
[5, 9]	0.004563004	0.054316652	0.055985061	0.015579510
[5,10]	0.004167736	0.049459625	0.050765734	0.013957072
[5,11]	0.004366686	0.045840288	0.046710569	0.012551566
[5,12]	0.005735011	0.042502640	0.042995343	0.011269484
[6, 6]	0.011898325	0.093208688	0.095250315	0.025953195
[6, 7]	0.005936090	0.077682553	0.080028605	0.022356878
[6, 8]	0.003989759	0.066358662	0.068785709	0.019549730
[6, 9]	0.003284189	0.058571692	0.060796164	0.017334071
[6,10]	0.003038982	0.053339126	0.055140993	0.015535720
[6,11]	0.003252934	0.049438133	0.050742879	0.013975450
[7, 7]	0.005015610	0.083564795	0.086710278	0.024805714
[7, 8]	0.003503421	0.071483086	0.074621008	0.021706909
[7, 9]	0.002942148	0.063155719	0.066014690	0.019257415
[7,10]	0.002773366	0.057540293	0.059905949	0.017266077
[8, 8]	0.003557641	0.078144937	0.081957174	0.024173162
[8, 9]	0.003066944	0.069221220	0.072656535	0.021464960

[R, S]	DISTRIBUTION (4,2)	DISTRIBUTION (4,3)	DISTRIBUTION (4,4)	DISTRIBUTION (4,5)
[1, 1]	2.196548718	1.392021099	0.874992454	0.801629570
[1, 2]	0.553519431	0.389473871	0.279005936	0.277698517
[1, 3]	0.193286922	0.176134338	0.150853633	0.150535275
[1, 4]	0.103596324	0.111390296	0.103008399	0.102613897
[1, 5]	0.076317643	0.083904590	0.078897552	0.078568339
[1, 6]	0.062776393	0.067859819	0.064441412	0.064188882
[1, 7]	0.053306927	0.056768133	0.054634557	0.054443719
[1, 8]	0.046091132	0.048701114	0.047399229	0.047251819
[1, 9]	0.040552653	0.042804868	0.041813634	0.041692928
[1,10]	0.036226566	0.038450295	0.037366147	0.037260138
[1,11]	0.032641038	0.035073656	0.033718867	0.033623326
[1,12]	0.029483693	0.032271389	0.030723575	0.030644315
[1,13]	0.027195444	0.030069401	0.028432507	0.028388997
[1,14]	0.028573843	0.029506962	0.027050422	0.027092958
[1,15]	0.040150959	0.033325499	0.027151924	0.027365619
[1,16]	0.063908456	0.043668282	0.030017319	0.029599100
[2, 2]	0.441545932	0.320536690	0.252133022	0.260465579
[2, 3]	0.142841408	0.139511071	0.135682539	0.139602283
[2, 4]	0.069935519	0.086309174	0.092664843	0.094836254
[2, 5]	0.049454499	0.064455807	0.071041166	0.072551570
[2, 6]	0.040239358	0.051980385	0.058070782	0.059262427
[2, 7]	0.034038514	0.043434501	0.04925[illegible]63	0.050266889
[2, 8]	0.029350862	0.037237508	0.042761398	0.043632884
[2, 9]	0.025760346	0.032711694	0.037739241	0.038506190
[2,10]	0.022965097	0.029370582	0.033737514	0.034417332
[2,11]	0.020662546	0.026782425	0.030453867	0.031062324
[2,12]	0.018646207	0.024637974	0.027757299	0.028314815
[2,13]	0.017176151	0.022955126	0.025696305	0.026235954
[2,14]	0.017987668	0.022524652	0.024455894	0.025043186
[2,15]	0.025194480	0.025435970	0.024555991	0.025299919
[3, 3]	0.104967761	0.119363968	0.132193707	0.135250558
[3, 4]	0.049724872	0.074447694	0.090903538	0.092466658
[3, 5]	0.034503450	0.055879470	0.069985603	0.071030914
[3, 6]	0.028007279	0.045222036	0.057361281	0.058174915
[3, 7]	0.023740145	0.037898846	0.048746484	0.049430558
[3, 8]	0.020528913	0.032581468	0.042366654	0.042961630
[3, 9]	0.018066376	0.028692778	0.037428923	0.037952494
[3,10]	0.016139647	0.025812513	0.033487261	0.033950129
[3,11]	0.014540892	0.023569092	0.030248071	0.030661331
[3,12]	0.013136028	0.021702023	0.027587324	0.027967604
[3,13]	0.012130406	0.020241790	0.025556474	0.025932676
[3,14]	0.012781509	0.019901268	0.024339898	0.024771960
[4, 4]	0.044099891	0.073361895	0.094018209	0.095272348
[4, 5]	0.031446344	0.055680833	0.072708601	0.073536292
[4, 6]	0.025747500	0.045294927	0.059756045	0.060401274
[4, 7]	0.021938082	0.038086974	0.050872809	0.051419048
[4, 8]	0.019059440	0.032834563	0.044273688	0.044751851
[4, 9]	0.016845300	0.028986108	0.039155655	0.039577812
[4,10]	0.015100754	0.026126605	0.035062012	0.035435265
[4,11]	0.013636650	0.023887433	0.031692520	0.032025768
[4,12]	0.012339900	0.022014991	0.028924027	0.029232661
[4,13]	0.011429482	0.020553648	0.026814078	0.027126347
[5, 5]	0.033231120	0.058958579	0.077673287	0.078456028
[5, 6]	0.027468468	0.048136663	0.064009622	0.064624972
[5, 7]	0.023478138	0.040559544	0.054588631	0.055113521
[5, 8]	0.020442115	0.035021628	0.047568980	0.048030953
[5, 9]	0.018101919	0.030958281	0.042114442	0.042523562
[5,10]	0.016252250	0.027933534	0.037743457	0.038105654
[5,11]	0.014692164	0.025558099	0.034139875	0.034463683
[5,12]	0.013305430	0.023566779	0.031178075	0.031479384
[6, 6]	0.030193534	0.051945201	0.069681561	0.070331231
[6, 7]	0.025850318	0.043830191	0.059524389	0.060081881
[6, 8]	0.022527823	0.037887401	0.051931652	0.052424038
[6, 9]	0.019963111	0.033522056	0.046020833	0.046457836
[6,10]	0.017933029	0.030267757	0.041276465	0.041663956
[6,11]	0.016217726	0.027707148	0.037359795	0.037706807
[7, 7]	0.028514848	0.047695666	0.065536987	0.066155907
[7, 8]	0.024872409	0.041284626	0.057244297	0.057791949
[7, 9]	0.022056370	0.036568890	0.050775239	0.051261864
[7,10]	0.019823289	0.033046215	0.045574425	0.046006451
[8, 8]	0.027621604	0.045552471	0.063655522	0.064271743
[8, 9]	0.024521757	0.040417235	0.056521955	0.057069702

[R, S]	DISTRIBUTION (4,6)	DISTRIBUTION (4,7)	DISTRIBUTION (4,8)	DISTRIBUTION (5,1)
[1, 1]	0.712267705	0.511987122	0.213403976	2.615008078
[1, 2]	0.243276574	0.200390798	0.121501506	0.258173372
[1, 3]	0.139053786	0.123213060	0.086130621	0.106963665
[1, 4]	0.098095281	0.089939639	0.067467570	0.085482361
[1, 5]	0.076224175	0.071081192	0.055837188	0.073141985
[1, 6]	0.062532081	0.058778538	0.047666267	0.063087110
[1, 7]	0.053050709	0.050041228	0.041445534	0.054719303
[1, 8]	0.046025465	0.043487078	0.036458238	0.047816353
[1, 9]	0.040612592	0.038404626	0.032306016	0.042151551
[1,10]	0.036315655	0.034345851	0.028733954	0.037406950
[1,11]	0.032792669	0.030983868	0.025562827	0.033187980
[1,12]	0.029851903	0.028104270	0.022662674	0.029101373
[1,13]	0.027434929	0.025585306	0.019947356	0.024827911
[1,14]	0.025578118	0.023364661	0.017347388	0.020270851
[1,15]	0.024655932	0.021549490	0.014662709	0.017292892
[1,16]	0.025700889	0.020438304	0.011091767	0.031595641
[2, 2]	0.230492434	0.207663604	0.160125853	0.149492052
[2, 3]	0.131871376	0.128316876	0.113599083	0.056160448
[2, 4]	0.093251395	0.093969008	0.089029223	0.044351455
[2, 5]	0.072607861	0.074426303	0.073708301	0.038071350
[2, 6]	0.059654251	0.061637932	0.062939239	0.032971785
[2, 7]	0.050663563	0.052533604	0.054737116	0.028723540
[2, 8]	0.043991591	0.045692910	0.048158899	0.025215620
[2, 9]	0.038845189	0.040381727	0.042680321	0.022324711
[2,10]	0.034754921	0.036134769	0.037965800	0.019878927
[2,11]	0.031397649	0.032612310	0.033779310	0.017673205
[2,12]	0.028593667	0.029592405	0.029949734	0.015510117
[2,13]	0.026289021	0.026949053	0.026363737	0.013233202
[2,14]	0.024519337	0.024617819	0.022929629	0.010808043
[2,15]	0.023645508	0.022713315	0.019382592	0.009296026
[3, 3]	0.134042482	0.137070253	0.133617189	0.061679947
[3, 4]	0.095404211	0.100842479	0.104879744	0.050485048
[3, 5]	0.074562498	0.080083125	0.086918468	0.043522533
[3, 6]	0.061408606	0.066439711	0.074270892	0.037782950
[3, 7]	0.052239252	0.056696940	0.064625690	0.032991542
[3, 8]	0.045414906	0.049361377	0.056882432	0.029032122
[3, 9]	0.040140766	0.043657484	0.050428158	0.025762785
[3,10]	0.035941614	0.039089908	0.044869946	0.022983064
[3,11]	0.032489546	0.035296178	0.039931110	0.020457391
[3,12]	0.029603950	0.032040328	0.035411039	0.017963385
[3,13]	0.027231893	0.029188480	0.031177060	0.015327808
[3,14]	0.025411494	0.026672275	0.027121259	0.012520704
[4, 4]	0.101105899	0.109468382	0.119946321	0.061682179
[4, 5]	0.079271991	0.087126104	0.099542199	0.053237897
[4, 6]	0.065418625	0.072387299	0.085134941	0.046225576
[4, 7]	0.055728054	0.061836967	0.074127515	0.040368626
[4, 8]	0.048499435	0.053880591	0.065278878	0.035528135
[4, 9]	0.042903726	0.047686254	0.057895262	0.031530993
[4,10]	0.038441516	0.042719646	0.051530863	0.028131828
[4,11]	0.034768124	0.038589506	0.045871206	0.025042231
[4,12]	0.031695427	0.035041911	0.040688198	0.021990185
[4,13]	0.029169388	0.031932731	0.035831234	0.018764058
[5, 5]	0.085504328	0.095077021	0.112642705	0.062402163
[5, 6]	0.070698570	0.079104271	0.096427132	0.054183032
[5, 7]	0.060306192	0.067644035	0.084014078	0.047316831
[5, 8]	0.052538340	0.058988965	0.074022047	0.041641786
[5, 9]	0.046516504	0.052242964	0.065675425	0.036955290
[5,10]	0.041707195	0.046827227	0.058474416	0.032970104
[5,11]	0.037742534	0.042318023	0.052065829	0.029348346
[5,12]	0.034423864	0.038441429	0.046193407	0.025771109
[6, 6]	0.077121917	0.086716046	0.108430341	0.061681954
[6, 7]	0.065876464	0.074236125	0.094532646	0.053872555
[6, 8]	0.057451404	0.064795019	0.083330726	0.047416557
[6, 9]	0.050910220	0.057427191	0.073963688	0.042083387
[6,10]	0.045678524	0.051504538	0.065875100	0.037546642
[6,11]	0.041359961	0.046566937	0.058671079	0.033422441
[7, 7]	0.072509508	0.081855917	0.105965514	0.060464180
[7, 8]	0.063307705	0.071519020	0.093459676	0.053237993
[7, 9]	0.056150443	0.063439355	0.082990484	0.047263827
[7,10]	0.050417013	0.056934676	0.073941382	0.042176325
[8, 8]	0.070369755	0.079542023	0.104798065	0.059603112
[8, 9]	0.062482007	0.070629283	0.093107145	0.052945016

[R, S]	DISTRIBUTION (5,2)	DISTRIBUTION (6,1)	DISTRIBUTION (6,2)	DISTRIBUTION (6,3)
[1, 1]	0.634224444	2.074944022	1.338089514	0.805047433
[1, 2]	0.190099862	0.778289208	0.513247744	0.316259975
[1, 3]	0.114904146	0.289857768	0.249976303	0.177657753
[1, 4]	0.086537684	0.101687861	0.136664415	0.118655158
[1, 5]	0.070790038	0.041949365	0.081693729	0.085413154
[1, 6]	0.060098243	0.024731199	0.051937474	0.062669012
[1, 7]	0.052051962	0.018779534	0.035115164	0.046499865
[1, 8]	0.045651803	0.015751178	0.026154244	0.036190500
[1, 9]	0.040370063	0.013815829	0.022318174	0.031147061
[1,10]	0.035865072	0.012562309	0.021982464	0.030009375
[1,11]	0.031885690	0.012086517	0.024159371	0.030859596
[1,12]	0.028250409	0.013571815	0.028153438	0.031915059
[1,13]	0.024865470	0.020699430	0.033666324	0.032264324
[1,14]	0.021807764	0.039167438	0.041027279	0.032191796
[1,15]	0.019549390	0.067462657	0.049659885	0.032595483
[1,16]	0.020123281	0.083227191	0.054098609	0.032780300
[2, 2]	0.175713086	0.813181510	0.512044892	0.320546140
[2, 3]	0.108320010	0.303518108	0.245411767	0.178442800
[2, 4]	0.082403843	0.102929579	0.132382728	0.118661895
[2, 5]	0.067746946	0.039365527	0.078404723	0.085256523
[2, 6]	0.057683767	0.021623646	0.049504781	0.062515492
[2, 7]	0.050062391	0.015905601	0.033196931	0.046373330
[2, 8]	0.043975323	0.013166517	0.024463659	0.036071719
[2, 9]	0.038936046	0.011452264	0.020648073	0.031014577
[2,10]	0.034625565	0.010340214	0.020171580	0.029853974
[2,11]	0.030808167	0.009863254	0.022079574	0.030684862
[2,12]	0.027313943	0.010902800	0.025709257	0.031731385
[2,13]	0.024057216	0.016355110	0.030762450	0.032080833
[2,14]	0.021117564	0.030953956	0.037524546	0.032010363
[2,15]	0.018956676	0.053979753	0.045490249	0.032411426
[3, 3]	0.117885012	0.255347159	0.216241064	0.175362282
[3, 4]	0.090493230	0.085333924	0.114960695	0.116679282
[3, 5]	0.074706909	0.029362445	0.067307953	0.083827995
[3, 6]	0.063759565	0.014165493	0.042196242	0.061457022
[3, 7]	0.055423770	0.009784303	0.028108830	0.045601354
[3, 8]	0.048743583	0.007929500	0.020541231	0.035511287
[3, 9]	0.043199229	0.006824827	0.017184276	0.030583680
[3,10]	0.038446068	0.006113057	0.016674328	0.029478175
[3,11]	0.034228164	0.005782137	0.018188436	0.030314145
[3,12]	0.030361368	0.006302356	0.021160950	0.031346495
[3,13]	0.026754645	0.009300748	0.025329615	0.031685505
[3,14]	0.023500840	0.017566058	0.030918905	0.031613137
[4, 4]	0.102123846	0.062557595	0.097530946	0.116902129
[4, 5]	0.084544911	0.020247686	0.056626323	0.084084267
[4, 6]	0.072268141	0.008478341	0.035285607	0.061627511
[4, 7]	0.062885625	0.005377155	0.023398451	0.045683090
[4, 8]	0.055349521	0.004255034	0.017015890	0.035541771
[4, 9]	0.049084180	0.003639393	0.014161959	0.030602269
[4,10]	0.043704990	0.003250679	0.013685851	0.029510342
[4,11]	0.038925321	0.003071191	0.014897058	0.030368134
[4,12]	0.034539141	0.003349164	0.017323490	0.031416683
[4,13]	0.030445970	0.004939588	0.020741722	0.031760608
[5, 5]	0.095538447	0.014165003	0.048305163	0.084186122
[5, 6]	0.081753285	0.005683287	0.029990223	0.061695510
[5, 7]	0.071190968	0.003443189	0.019808617	0.045638137
[5, 8]	0.062694091	0.002703817	0.014347346	0.035365596
[5, 9]	0.055621695	0.002325859	0.011894301	0.030317492
[5,10]	0.049543300	0.002093412	0.011457704	0.029160033
[5,11]	0.044137425	0.001998598	0.012447944	0.029997249
[5,12]	0.039173188	0.002223098	0.014466718	0.031055659
[6, 6]	0.091898858	0.004570978	0.025957967	0.060572588
[6, 7]	0.080076670	0.002889353	0.017128962	0.044788977
[6, 8]	0.070554055	0.002306334	0.012391463	0.034589084
[6, 9]	0.062619919	0.002006797	0.010265150	0.029486280
[6,10]	0.055794640	0.001824415	0.009881112	0.028224543
[6,11]	0.049719566	0.001762236	0.010722783	0.028974265
[7, 7]	0.089713466	0.002859232	0.015329775	0.043661466
[7, 8]	0.079088006	0.002337248	0.011194888	0.033868814
[7, 9]	0.070225280	0.002050867	0.009367467	0.028898059
[7,10]	0.062593551	0.001875431	0.009083786	0.027589133
[8, 8]	0.088644445	0.002529148	0.011023395	0.034699185
[8, 9]	0.078753184	0.002233414	0.009477633	0.030088839

[R, S]	DISTRIBUTION (6,4)	DISTRIBUTION (6,5)	DISTRIBUTION (6,6)	DISTRIBUTION (7,1)
[1, 1]	0.662364513	0.442794342	0.338582969	3.092467681
[1, 2]	0.271015433	0.199960832	0.168212663	0.726586452
[1, 3]	0.158795405	0.127350658	0.110561714	0.203574206
[1, 4]	0.110186806	0.093535937	0.083172267	0.082806694
[1, 5]	0.081891181	0.072684988	0.066424164	0.053809302
[1, 6]	0.061942494	0.057630348	0.054439071	0.042892531
[1, 7]	0.047354784	0.046376684	0.045389037	0.036074067
[1, 8]	0.037715278	0.038508120	0.038724975	0.031052128
[1, 9]	0.032657072	0.033704602	0.034102467	0.027279545
[1,10]	0.031016890	0.031169287	0.030971003	0.024402507
[1,11]	0.031048415	0.029750448	0.028614557	0.022094693
[1,12]	0.031151096	0.028420511	0.026418104	0.020238004
[1,13]	0.030539272	0.026661035	0.024082036	0.019762134
[1,14]	0.029444037	0.024509924	0.021646099	0.025135370
[1,15]	0.028657970	0.022308131	0.019286155	0.046339306
[1,16]	0.027732060	0.019781718	0.016219560	0.085074449
[2, 2]	0.282531344	0.224025534	0.199978776	0.581692680
[2, 3]	0.164584054	0.142772381	0.131209923	0.152301531
[2, 4]	0.113928444	0.104948074	0.098663379	0.053465737
[2, 5]	0.084586481	0.081592233	0.078795747	0.031451980
[2, 6]	0.063964781	0.064720309	0.064591422	0.024370515
[2, 7]	0.048911006	0.052118610	0.053874610	0.020318387
[2, 8]	0.038971295	0.043322248	0.045988709	0.017391025
[2, 9]	0.033755722	0.037962042	0.040519327	0.015202286
[2,10]	0.032061831	0.035134192	0.036810410	0.013543756
[2,11]	0.032089884	0.033544151	0.034013793	0.012227708
[2,12]	0.032191222	0.032043555	0.031403428	0.011173198
[2,13]	0.031556853	0.030057342	0.028627086	0.010853397
[2,14]	0.030425606	0.027632798	0.025733555	0.013654941
[2,15]	0.029614726	0.025154022	0.022931135	0.025069474
[3, 3]	0.167449274	0.155283929	0.145728535	0.105270236
[3, 4]	0.116185321	0.114475756	0.109897192	0.033170743
[3, 5]	0.086334653	0.089114565	0.087898579	0.017520308
[3, 6]	0.065302368	0.070734683	0.072120568	0.013173137
[3, 7]	0.049962836	0.057004890	0.060209363	0.010940447
[3, 8]	0.039869728	0.047440443	0.051452853	0.009367459
[3, 9]	0.034607713	0.041630588	0.045385524	0.008194669
[3,10]	0.032926420	0.038572872	0.041266988	0.007304370
[3,11]	0.032976892	0.036845940	0.038148621	0.006596495
[3,12]	0.033079268	0.035199493	0.035225668	0.006031666
[3,13]	0.032419955	0.033014891	0.032113096	0.005875655
[3,14]	0.031255359	0.030351728	0.028871219	0.007432631
[4, 4]	0.120267384	0.124193594	0.121601800	0.024531900
[4, 5]	0.089488776	0.096772157	0.097397017	0.013038314
[4, 6]	0.067688718	0.076833468	0.079972145	0.009822455
[4, 7]	0.051757947	0.061921603	0.066801855	0.008210287
[4, 8]	0.041283022	0.051539192	0.057123613	0.007080728
[4, 9]	0.035840600	0.045244362	0.050423593	0.006236359
[4,10]	0.034122181	0.041940403	0.045874902	0.005589384
[4,11]	0.034196181	0.040075352	0.042422449	0.005067139
[4,12]	0.034313305	0.038289084	0.039176666	0.004649572
[4,13]	0.033630855	0.035912674	0.035716358	0.004567530
[5, 5]	0.092070460	0.103426211	0.106685280	0.012545020
[5, 6]	0.069637856	0.082116671	0.087635078	0.009719330
[5, 7]	0.053166023	0.066145910	0.073217934	0.008185927
[5, 8]	0.042283852	0.055008055	0.062620065	0.007093278
[5, 9]	0.036598926	0.048250495	0.055285513	0.006272769
[5,10]	0.034786219	0.044708197	0.050307580	0.005640397
[5,11]	0.034856764	0.042718342	0.046527741	0.005125281
[5,12]	0.034994454	0.040819012	0.042970766	0.004712483
[6, 6]	0.070160963	0.086186248	0.094812396	0.010457010
[6, 7]	0.053553401	0.069431973	0.079246266	0.008850870
[6, 8]	0.042485350	0.057706815	0.067791888	0.007683664
[6, 9]	0.036624916	0.050565026	0.059857654	0.006804130
[6,10]	0.034694327	0.046810388	0.054469632	0.006124648
[6,11]	0.034716868	0.044707832	0.050377696	0.005569598
[7, 7]	0.053545598	0.072538401	0.085462908	0.009698925
[7, 8]	0.042629300	0.060416896	0.073204320	0.008431359
[7, 9]	0.036767705	0.052987523	0.064691582	0.007473339
[7,10]	0.034757345	0.049042241	0.058889993	0.006731629
[8, 8]	0.044397711	0.064786167	0.080114305	0.009338781
[8, 9]	0.038743415	0.057125401	0.070973013	0.008288815

[R, S]	DISTRIBUTION (7,2)	DISTRIBUTION (7,3)	DISTRIBUTION (7,4)	DISTRIBUTION (7,5)
[1, 1]	1.919574780	1.624573919	1.416309646	1.030831787
[1, 2]	0.688107609	0.524707478	0.522173894	0.361922009
[1, 3]	0.211964741	0.224274339	0.224422001	0.184416609
[1, 4]	0.085067231	0.121792311	0.121351517	0.118553335
[1, 5]	0.057351954	0.079123606	0.078671134	0.084536414
[1, 6]	0.049517913	0.058002779	0.057658519	0.063871968
[1, 7]	0.044672216	0.046115842	0.045870859	0.050713061
[1, 8]	0.040174371	0.038731325	0.038558533	0.042277113
[1, 9]	0.035709147	0.033919193	0.033793800	0.036977943
[1,10]	0.031254570	0.030822646	0.030730350	0.033837856
[1,11]	0.026875389	0.029075978	0.029022021	0.032204630
[1,12]	0.023015864	0.028744736	0.028766681	0.031593046
[1,13]	0.021793685	0.030378713	0.030580837	0.031609456
[1,14]	0.029659001	0.035245004	0.035892984	0.032221684
[1,15]	0.054416718	0.045513854	0.046870703	0.034686901
[1,16]	0.074566981	0.057697156	0.056240546	0.038785229
[2, 2]	0.773125176	0.477996971	0.519833265	0.343681840
[2, 3]	0.231966708	0.195314454	0.212308274	0.169630165
[2, 4]	0.078194656	0.102336998	0.109639037	0.107471034
[2, 5]	0.045079914	0.065072845	0.069098767	0.076264277
[2, 6]	0.037309098	0.047048796	0.049749275	0.057484419
[2, 7]	0.033457001	0.037080659	0.039139616	0.045561529
[2, 8]	0.030069748	0.030969624	0.032669328	0.037929434
[2, 9]	0.026726171	0.027014862	0.028491009	0.033138828
[2,10]	0.023391150	0.024474383	0.025810318	0.030299861
[2,11]	0.020105985	0.023030978	0.024300256	0.028821077
[2,12]	0.017171886	0.022718511	0.024018667	0.028264014
[2,13]	0.016063616	0.023961814	0.025467141	0.028273866
[2,14]	0.021415156	0.027759508	0.029834021	0.028819096
[2,15]	0.039198446	0.035838230	0.038950788	0.031020704
[3, 3]	0.181477400	0.157183823	0.170861175	0.155608596
[3, 4]	0.055142964	0.080636995	0.086032520	0.098667336
[3, 5]	0.026576165	0.050765603	0.053516374	0.070133778
[3, 6]	0.020584118	0.036558748	0.038316059	0.052973277
[3, 7]	0.018266226	0.028759120	0.030062151	0.042068969
[3, 8]	0.016398415	0.023998921	0.025058800	0.035082650
[3, 9]	0.014573665	0.020926648	0.021839430	0.030695994
[3,10]	0.012755278	0.018955740	0.019777528	0.028097922
[3,11]	0.010965491	0.017837344	0.018617169	0.026747679
[3,12]	0.009372368	0.017596574	0.018400423	0.026243063
[3,13]	0.008786799	0.018561144	0.019509591	0.026258017
[3,14]	0.011729589	0.021503689	0.022853867	0.026767915
[4, 4]	0.041002568	0.068169127	0.071900857	0.096020499
[4, 5]	0.019403641	0.043036776	0.044830699	0.068411092
[4, 6]	0.014672942	0.031102481	0.032211761	0.051743065
[4, 7]	0.012952586	0.024543611	0.025355565	0.041138565
[4, 8]	0.011620540	0.020531931	0.021190293	0.034339503
[4, 9]	0.010326913	0.017939531	0.018506756	0.030068947
[4,10]	0.009039093	0.016276986	0.016788813	0.027540040
[4,11]	0.007776235	0.015339156	0.015828529	0.026227128
[4,12]	0.006677545	0.015153999	0.015668615	0.025738338
[4,13]	0.006383983	0.016006144	0.016637438	0.025755767
[5, 5]	0.018930797	0.039641370	0.040996235	0.068039803
[5, 6]	0.015009758	0.028852383	0.029683979	0.051513304
[5, 7]	0.013330529	0.022881908	0.023492035	0.040991894
[5, 8]	0.011966101	0.019214212	0.019711859	0.034245808
[5, 9]	0.010634460	0.016839226	0.017270540	0.030009225
[5,10]	0.009308737	0.015316670	0.015708271	0.027501592
[5,11]	0.008011565	0.014465199	0.014843496	0.026201212
[5,12]	0.006899369	0.014320048	0.014725940	0.025718661
[6, 6]	0.017265272	0.028531641	0.029261951	0.052024395
[6, 7]	0.015454894	0.022766452	0.023308520	0.041484038
[6, 8]	0.013883628	0.019199745	0.019646106	0.034725359
[6, 9]	0.012339336	0.016883572	0.017273362	0.030482907
[6,10]	0.010801228	0.015399682	0.015756060	0.027974933
[6,11]	0.009296950	0.014578605	0.014926083	0.026678222
[7, 7]	0.017943487	0.023732564	0.024271761	0.043025142
[7, 8]	0.016128989	0.020104956	0.020553468	0.036145424
[7, 9]	0.014335625	0.017740223	0.018134633	0.031829558
[7,10]	0.012548735	0.016226470	0.016589317	0.029283831
[8, 8]	0.018424909	0.021831078	0.022313177	0.038965486
[8, 9]	0.016376885	0.019335647	0.019762092	0.034471659

[R, S]	DISTRIBUTION (7,6)	DISTRIBUTION (7,7)	DISTRIBUTION (7,8)	DISTRIBUTION (7,9)
[1, 1]	0.576064830	0.428159349	0.220145364	0.184586136
[1, 2]	0.226893590	0.183006914	0.124354509	0.111929347
[1, 3]	0.135883713	0.117685907	0.087626257	0.080223163
[1, 4]	0.097294758	0.087304946	0.068077988	0.063298107
[1, 5]	0.075325759	0.068967451	0.056067935	0.052886463
[1, 6]	0.060623676	0.056414050	0.047900455	0.045792769
[1, 7]	0.050112926	0.047366548	0.041822602	0.040447048
[1, 8]	0.042617933	0.040763711	0.036924828	0.035976120
[1, 9]	0.037459386	0.035945498	0.032738715	0.031936459
[1,10]	0.033979070	0.032386300	0.029035997	0.028175886
[1,11]	0.031487436	0.029621924	0.025718245	0.024707863
[1,12]	0.029400472	0.027255625	0.022748076	0.021588449
[1,13]	0.027396936	0.024994231	0.020081854	0.018813440
[1,14]	0.025492772	0.022692838	0.017595819	0.016266894
[1,15]	0.024056274	0.020388125	0.014977506	0.013667837
[1,16]	0.023296424	0.018193534	0.011372143	0.010024889
[2, 2]	0.232857067	0.201317269	0.163390415	0.153173595
[2, 3]	0.139291964	0.130279009	0.115213168	0.109551447
[2, 4]	0.099830928	0.096964069	0.089547807	0.086361798
[2, 5]	0.077362721	0.076745695	0.073771714	0.072128115
[2, 6]	0.062314462	0.062864071	0.063037987	0.062440194
[2, 7]	0.051555169	0.052845272	0.055046951	0.055143959
[2, 8]	0.043885508	0.045528133	0.048605301	0.049045124
[2, 9]	0.038607441	0.040184690	0.043098443	0.043537103
[2,10]	0.035043431	0.036232407	0.038226919	0.038411018
[2,11]	0.032485872	0.033156281	0.033861656	0.033684196
[2,12]	0.030337923	0.030516666	0.029953556	0.029432421
[2,13]	0.028273615	0.027989625	0.026445008	0.025649742
[2,14]	0.026312757	0.025416478	0.023172865	0.022178196
[2,15]	0.024836240	0.022840418	0.019725557	0.018634822
[3, 3]	0.145365338	0.143512245	0.134726791	0.129666387
[3, 4]	0.104661240	0.107144684	0.104838433	0.102376804
[3, 5]	0.081305520	0.084946939	0.086442816	0.085597959
[3, 6]	0.065592898	0.069662222	0.073909211	0.074154927
[3, 7]	0.054340048	0.058615954	0.064565617	0.065517610
[3, 8]	0.046318398	0.050542924	0.057025696	0.058284676
[3, 9]	0.040799551	0.044643880	0.050575515	0.051746563
[3,10]	0.037069549	0.040276248	0.044867345	0.045661051
[3,11]	0.034384273	0.036871272	0.039751521	0.040050650
[3,12]	0.032119721	0.033943821	0.035170955	0.035004664
[3,13]	0.029938912	0.031137270	0.031057666	0.030514302
[3,14]	0.027868249	0.028278185	0.027219528	0.026390735
[4, 4]	0.111627874	0.117434494	0.118984084	0.117725015
[4, 5]	0.086884214	0.093213873	0.098250472	0.098595225
[4, 6]	0.070173430	0.076501539	0.084087725	0.085506161
[4, 7]	0.058187773	0.064412021	0.073504956	0.075593296
[4, 8]	0.049642503	0.055572230	0.064949648	0.067270112
[4, 9]	0.043764671	0.049110098	0.057622266	0.059736427
[4,10]	0.039790083	0.044322289	0.051133606	0.052722699
[4,11]	0.036922779	0.040585692	0.045316583	0.046258271
[4,12]	0.034497869	0.037369074	0.040107266	0.040445135
[4,13]	0.032159138	0.034282508	0.035427649	0.035270398
[5, 5]	0.092888151	0.101300709	0.110744180	0.112603678
[5, 6]	0.075083547	0.083195030	0.094896680	0.097779750
[5, 7]	0.062297960	0.070088432	0.083019533	0.086506944
[5, 8]	0.053179485	0.060501144	0.073395921	0.077011399
[5, 9]	0.046907112	0.053489555	0.065141497	0.068402708
[5,10]	0.042664089	0.048291086	0.057825961	0.060385971
[5,11]	0.039599384	0.044229862	0.051265188	0.052999487
[5,12]	0.037003454	0.040729923	0.045388565	0.046358698
[6, 6]	0.080109502	0.090042144	0.106876455	0.111526566
[6, 7]	0.066523316	0.075921902	0.093578623	0.098745652
[6, 8]	0.056829796	0.065588268	0.082776960	0.087941760
[6, 9]	0.050158377	0.058025984	0.073497510	0.078129132
[6,10]	0.045640641	0.052413081	0.065266413	0.068987769
[6,11]	0.042371989	0.048021099	0.057881526	0.060567595
[7, 7]	0.071338470	0.082544084	0.105350600	0.112363366
[7, 8]	0.061045478	0.071404407	0.093239883	0.100106597
[7, 9]	0.053953750	0.063243379	0.082819750	0.088954450
[7,10]	0.049138681	0.057174708	0.073569188	0.078560227
[8, 8]	0.066732679	0.078743248	0.104888743	0.113315703
[8, 9]	0.059136188	0.069864840	0.093202464	0.100709577

[R, S]	DISTRIBUTION (8,1)	DISTRIBUTION (8,2)	DISTRIBUTION (8,3)	DISTRIBUTION (8,4)
[1, 1]	3.481633812	2.844308740	2.134729002	1.604749157
[1, 2]	0.552606265	0.532398912	0.428774180	0.358091487
[1, 3]	0.145118551	0.165059090	0.171856487	0.165531130
[1, 4]	0.083440382	0.094816922	0.107796911	0.109314707
[1, 5]	0.066615608	0.073984331	0.081846085	0.083359920
[1, 6]	0.056297311	0.062165519	0.066510713	0.067462246
[1, 7]	0.048263576	0.053254136	0.055751803	0.056354675
[1, 8]	0.041883638	0.046224386	0.047844455	0.048284366
[1, 9]	0.036840752	0.040679673	0.042011878	0.042408973
[1,10]	0.032782902	0.036223523	0.037660397	0.038093429
[1,11]	0.029305316	0.032411713	0.034238319	0.034763156
[1,12]	0.026032536	0.028867245	0.031320955	0.031975483
[1,13]	0.022854570	0.025638058	0.028848980	0.029595290
[1,14]	0.021218386	0.024559216	0.027775133	0.028149749
[1,15]	0.029515221	0.032976068	0.031851404	0.029804581
[1,16]	0.067529340	0.063846002	0.050267180	0.040962907
[2, 2]	0.372613679	0.365711876	0.297000802	0.265017345
[2, 3]	0.080314476	0.099853342	0.112134734	0.119668374
[2, 4]	0.038463320	0.051397531	0.068654726	0.078761190
[2, 5]	0.029504930	0.038791621	0.051772558	0.060083786
[2, 6]	0.024802429	0.032387627	0.042012661	0.048676033
[2, 7]	0.021232354	0.027696604	0.035216176	0.040711197
[2, 8]	0.018405854	0.024013160	0.030231780	0.034925098
[2, 9]	0.016172533	0.021109841	0.026555891	0.030710552
[2,10]	0.014376935	0.018779016	0.023811191	0.027609863
[2,11]	0.012842056	0.016790671	0.021650497	0.025210810
[2,12]	0.011402770	0.014948244	0.019808021	0.023198252
[2,13]	0.010009105	0.013273189	0.018249403	0.021480501
[2,14]	0.009291144	0.012708136	0.017581421	0.020446345
[2,15]	0.012910315	0.017040065	0.020180986	0.021673070
[3, 3]	0.054733866	0.070020605	0.091529497	0.107637714
[3, 4]	0.026074780	0.036438652	0.057367731	0.072106600
[3, 5]	0.019994281	0.027628537	0.043739664	0.055467717
[3, 6]	0.016893455	0.023165486	0.035699897	0.045140601
[3, 7]	0.014546217	0.019899058	0.030051805	0.037878717
[3, 8]	0.012685492	0.017332652	0.025896521	0.032589282
[3, 9]	0.011209218	0.015303327	0.022825137	0.028729925
[3,10]	0.010009492	0.013660350	0.020520928	0.025881118
[3,11]	0.008966511	0.012240671	0.018692496	0.023664352
[3,12]	0.007973266	0.010911490	0.017121951	0.021794373
[3,13]	0.007010047	0.009708646	0.015794573	0.020197000
[3,14]	0.006561063	0.009364602	0.015255004	0.019249711
[4, 4]	0.026336510	0.035571209	0.057482579	0.074085955
[4, 5]	0.020844968	0.027819087	0.044390244	0.057398087
[4, 6]	0.017717214	0.023503026	0.036423819	0.046875968
[4, 7]	0.015307111	0.020267479	0.030761600	0.039427734
[4, 8]	0.013390949	0.017714228	0.026579726	0.033989275
[4, 9]	0.011867204	0.015690015	0.023482518	0.030015884
[4,10]	0.010622290	0.014041580	0.021151296	0.027076309
[4,11]	0.009530507	0.012603760	0.019291139	0.024780045
[4,12]	0.008481962	0.011246453	0.017684834	0.022835514
[4,13]	0.007463127	0.010019946	0.016327265	0.021173182
[5, 5]	0.023553298	0.030925546	0.047635129	0.061429197
[5, 6]	0.020077434	0.026243239	0.039203843	0.050277946
[5, 7]	0.017359543	0.022661974	0.033164507	0.042348817
[5, 8]	0.015195285	0.019826837	0.028693047	0.036549510
[5, 9]	0.013473210	0.017576774	0.025377401	0.032308555
[5,10]	0.012064875	0.015741403	0.022877692	0.029166759
[5,11]	0.010827882	0.014136413	0.020877980	0.026707176
[5,12]	0.009638148	0.012617881	0.019147134	0.024620046
[6, 6]	0.022528938	0.029345427	0.042575906	0.054307469
[6, 7]	0.019487123	0.025357490	0.036057686	0.045795566
[6, 8]	0.017062436	0.022194350	0.031224465	0.039562688
[6, 9]	0.015132119	0.019682121	0.027637107	0.035000619
[6,10]	0.013552411	0.017631130	0.024928943	0.031616484
[6,11]	0.012163898	0.015835836	0.022758684	0.028962420
[7, 7]	0.021679064	0.028186538	0.039360928	0.049799691
[7, 8]	0.018992538	0.024685671	0.034126129	0.043079836
[7, 9]	0.016851629	0.021902180	0.030235733	0.038155177
[7,10]	0.015097028	0.019626320	0.027292917	0.034494656
[8, 8]	0.021169812	0.027508161	0.037704495	0.047508961
[8, 9]	0.018799364	0.024427535	0.033458150	0.042150851

[R, S]	DISTRIBUTION (8,5)	DISTRIBUTION (8,6)	DISTRIBUTION (9,1)	DISTRIBUTION (9,2)
[1, 1]	0.922238080	0.517747296	2.887475658	2.012582519
[1, 2]	0.259143289	0.186112143	0.431803787	0.348589908
[1, 3]	0.142942386	0.117038767	0.131286701	0.143549612
[1, 4]	0.102583372	0.087255443	0.091727085	0.100036468
[1, 5]	0.080468929	0.069845500	0.077089770	0.081591848
[1, 6]	0.065243008	0.058087905	0.066157077	0.069139147
[1, 7]	0.053988550	0.049501728	0.057197862	0.059423171
[1, 8]	0.045821883	0.042987412	0.049880266	0.051682895
[1, 9]	0.040187069	0.037951674	0.043937415	0.045524710
[1,10]	0.036401219	0.033958006	0.039015409	0.040525841
[1,11]	0.033659250	0.030631899	0.034682078	0.036205134
[1,12]	0.031223245	0.027688555	0.030512043	0.032127502
[1,13]	0.028677237	0.024958458	0.026215816	0.028101277
[1,14]	0.026177602	0.022375244	0.022327729	0.024730333
[1,15]	0.024892401	0.020054368	0.024755866	0.025421573
[1,16]	0.028276199	0.019069489	0.052759806	0.041350749
[2, 2]	0.221955734	0.188677213	0.305415011	0.228684166
[2, 3]	0.122835623	0.120325223	0.070651602	0.088232085
[2, 4]	0.088620427	0.090330450	0.042989926	0.060517438
[2, 5]	0.069726766	0.072590382	0.035627379	0.049281648
[2, 6]	0.056652739	0.060522380	0.030577957	0.041806946
[2, 7]	0.046984966	0.051671671	0.026468677	0.035992971
[2, 8]	0.039980178	0.044940369	0.023113630	0.031362569
[2, 9]	0.035151723	0.039727213	0.020384416	0.027672660
[2,10]	0.031900618	0.035582886	0.018115988	0.024665559
[2,11]	0.029527886	0.032121051	0.016110467	0.022052532
[2,12]	0.027400278	0.029049825	0.014174776	0.019576429
[2,13]	0.025169526	0.026196887	0.012179621	0.017130895
[2,14]	0.022986335	0.023495901	0.010387590	0.015100147
[2,15]	0.021883924	0.021070922	0.011594136	0.015588876
[3, 3]	0.125052102	0.132250500	0.057304809	0.078560645
[3, 4]	0.091297008	0.099773409	0.036450585	0.056174549
[3, 5]	0.072177726	0.080385677	0.030437610	0.046304838
[3, 6]	0.058792799	0.067130192	0.026255102	0.039477397
[3, 7]	0.048868195	0.057380779	0.022839961	0.034113299
[3, 8]	0.041684887	0.049954124	0.020047412	0.029830589
[3, 9]	0.036740095	0.044195420	0.017766676	0.026408047
[3,10]	0.033405778	0.039610290	0.015851568	0.023600150
[3,11]	0.030954810	0.035773025	0.014132020	0.021135461
[3,12]	0.028736011	0.032363292	0.012448420	0.018779145
[3,13]	0.026400358	0.029192954	0.010701513	0.016446157
[3,14]	0.024119705	0.026190402	0.009155926	0.014531307
[4, 4]	0.099045905	0.110488401	0.041797969	0.062392681
[4, 5]	0.078523732	0.089175514	0.035301910	0.051865735
[4, 6]	0.064042752	0.074555205	0.030504261	0.044335653
[4, 7]	0.053282390	0.063781652	0.026567533	0.038372844
[4, 8]	0.045496639	0.055565995	0.023346048	0.033603688
[4, 9]	0.040141554	0.049189860	0.020712707	0.029787836
[4,10]	0.036529643	0.044107230	0.018496765	0.026649155
[4,11]	0.033867001	0.039847699	0.016500239	0.023882916
[4,12]	0.031446216	0.036058385	0.014538890	0.021228487
[4,13]	0.028892475	0.032532786	0.012500264	0.018597027
[5, 5]	0.085318545	0.098431906	0.041020515	0.059025563
[5, 6]	0.069622708	0.082376851	0.035464062	0.050508178
[5, 7]	0.057940842	0.070527065	0.030891003	0.043737501
[5, 8]	0.049485692	0.061482351	0.027147733	0.038317406
[5, 9]	0.043670750	0.054457212	0.024087398	0.033978708
[5,10]	0.039748666	0.048851275	0.021511677	0.030407299
[5,11]	0.036855978	0.044147255	0.019190465	0.027256238
[5,12]	0.034223953	0.039958195	0.016909670	0.024229618
[6, 6]	0.074887025	0.090723208	0.040227382	0.056749882
[6, 7]	0.062351426	0.077737458	0.035046216	0.049159810
[6, 8]	0.053272647	0.067815822	0.030803846	0.043080274
[6, 9]	0.047024409	0.060102633	0.027334192	0.038211159
[6,10]	0.042805640	0.053940485	0.024412708	0.034200385
[6,11]	0.039690692	0.048762915	0.021778915	0.030659110
[7, 7]	0.067009265	0.085759771	0.039242904	0.054758888
[7, 8]	0.057336608	0.074883118	0.034506959	0.048013269
[7, 9]	0.050670638	0.066416702	0.030630342	0.042605423
[7,10]	0.046155433	0.059642086	0.027362339	0.038144637
[8, 8]	0.062675628	0.083233037	0.038587112	0.053556406
[8, 9]	0.055535120	0.073901337	0.034274075	0.047562140

[R, S]	DISTRIBUTION (9,3)	DISTRIBUTION (9,4)	DISTRIBUTION (10,1)	DISTRIBUTION (10,2)
[1, 1]	1.117753915	0.836146735	3.226389983	0.352678251
[1, 2]	0.260760268	0.199257340	0.861162506	0.165670751
[1, 3]	0.136144967	0.118358496	0.225084126	0.110278192
[1, 4]	0.097720744	0.088176728	0.066013744	0.083678431
[1, 5]	0.078646791	0.071652828	0.031270221	0.066791164
[1, 6]	0.066326664	0.060754074	0.022316774	0.054525595
[1, 7]	0.057091012	0.052574382	0.018265316	0.045277570
[1, 8]	0.049786511	0.046036911	0.015593380	0.038523881
[1, 9]	0.043928972	0.040682651	0.013678715	0.033901686
[1,10]	0.039114534	0.036171700	0.012287505	0.030836083
[1,11]	0.034929217	0.032184696	0.011283540	0.028583676
[1,12]	0.031065672	0.028511632	0.010902825	0.026490952
[1,13]	0.027497723	0.025142557	0.013044605	0.024190783
[1,14]	0.024604540	0.022203781	0.024363570	0.021616387
[1,15]	0.023566163	0.019938453	0.055939502	0.018935366
[1,16]	0.029240907	0.021496014	0.099222704	0.016113680
[2, 2]	0.204810559	0.171854772	0.758842188	0.191912654
[2, 3]	0.106405955	0.105319946	0.195612873	0.128236418
[2, 4]	0.077004800	0.079607643	0.051331966	0.097532754
[2, 5]	0.062307021	0.065195725	0.020525832	0.077954817
[2, 6]	0.052696534	0.055509253	0.013525457	0.063701743
[2, 7]	0.045448652	0.048155717	0.010827224	0.052952936
[2, 8]	0.039704025	0.042248825	0.009149330	0.045111748
[2, 9]	0.035088494	0.037396501	0.007961950	0.039750565
[2,10]	0.031281583	0.033292979	0.007106132	0.036190782
[2,11]	0.027957423	0.029650634	0.006491761	0.033563079
[2,12]	0.024880055	0.026286570	0.006225218	0.031109665
[2,13]	0.022039674	0.023201285	0.007313725	0.028408845
[2,14]	0.019746795	0.020513279	0.013440791	0.025387498
[2,15]	0.018952454	0.018446457	0.031055534	0.022243997
[3, 3]	0.106514709	0.114774462	0.140864596	0.144893402
[3, 4]	0.078392214	0.087440582	0.034206747	0.110551789
[3, 5]	0.063892284	0.071878569	0.010993298	0.088494432
[3, 6]	0.054225401	0.061321787	0.006312453	0.072380454
[3, 7]	0.046869618	0.053265908	0.004882100	0.060217655
[3, 8]	0.041021148	0.046779777	0.004086760	0.051351215
[3, 9]	0.036311972	0.041443044	0.003536414	0.045295326
[3,10]	0.032413775	0.036920316	0.003142171	0.041272351
[3,11]	0.028994118	0.032896781	0.002860733	0.038292133
[3,12]	0.025818479	0.029176095	0.002733834	0.035497739
[3,13]	0.022888334	0.025764411	0.003188786	0.032416567
[3,14]	0.020533377	0.022794047	0.005812575	0.028970449
[4, 4]	0.086172125	0.097952620	0.022747557	0.123349917
[4, 5]	0.070574586	0.080740909	0.006570109	0.098819438
[4, 6]	0.060031617	0.068983775	0.003356728	0.080858223
[4, 7]	0.051961709	0.059979728	0.002527958	0.067290462
[4, 8]	0.045532924	0.052719543	0.002119657	0.057401553
[4, 9]	0.040349586	0.046739086	0.001843943	0.050651365
[4,10]	0.036048784	0.041662004	0.001646096	0.046167966
[4,11]	0.032264196	0.037136541	0.001504732	0.042842733
[4,12]	0.028742073	0.032947233	0.001448669	0.039719177
[4,13]	0.025492569	0.029106430	0.001722061	0.036272087
[5, 5]	0.079159202	0.090966289	0.004896818	0.108184517
[5, 6]	0.067436995	0.077822718	0.002602286	0.088536075
[5, 7]	0.058423386	0.067719347	0.001986215	0.073682290
[5, 8]	0.051231915	0.059560893	0.001684289	0.062852023
[5, 9]	0.045428708	0.052834385	0.001480200	0.055459604
[5,10]	0.040607123	0.047116542	0.001332495	0.050551375
[5,11]	0.036356697	0.042012296	0.001226353	0.046912071
[5,12]	0.032396045	0.037282977	0.001192127	0.043493100
[6, 6]	0.075512886	0.087490189	0.002531952	0.095448159
[6, 7]	0.065461123	0.076181312	0.001994246	0.079462535
[6, 8]	0.057431221	0.067036148	0.001704949	0.067795476
[6, 9]	0.050946379	0.059489707	0.001505620	0.059823978
[6,10]	0.045553268	0.053068705	0.001360418	0.054527595
[6,11]	0.040794013	0.047331130	0.001255893	0.050600169
[7, 7]	0.072980479	0.085290243	0.002137666	0.085393504
[7, 8]	0.064065681	0.075090705	0.001837201	0.072956897
[7, 9]	0.056858339	0.066665087	0.001625981	0.064438084
[7,10]	0.050856294	0.059487870	0.001471459	0.058756523
[8, 8]	0.071537400	0.084065846	0.002022798	0.079681527
[8, 9]	0.063536184	0.074676881	0.001794029	0.070568165

[R, S]	DISTRIBUTION (11,1)	DISTRIBUTION (11,2)	DISTRIBUTION (11,3)
[1, 1]	3.992238810	0.607768199	0.527903486
[1, 2]	0.505116615	0.189778711	0.181098096
[1, 3]	0.120228991	0.118644284	0.114979924
[1, 4]	0.075629955	0.088047427	0.086103654
[1, 5]	0.062108951	0.070244565	0.069077419
[1, 6]	0.052707617	0.058330650	0.057520054
[1, 7]	0.045224410	0.049698422	0.049055754
[1, 8]	0.039252119	0.043166992	0.042619336
[1, 9]	0.034514904	0.038107872	0.037630886
[1,10]	0.030690775	0.034082035	0.033662794
[1,11]	0.027404507	0.030730281	0.030348154
[1,12]	0.024292935	0.027788201	0.027406048
[1,13]	0.021125694	0.025094775	0.024662851
[1,14]	0.018582314	0.022563133	0.022036792
[1,15]	0.023729218	0.020237838	0.019595043
[1,16]	0.062827562	0.019625676	0.018530960
[2, 2]	0.334954625	0.186213068	0.183306838
[2, 3]	0.058346700	0.118986536	0.118561892
[2, 4]	0.029315326	0.089107060	0.089502476
[2, 5]	0.023391292	0.071439335	0.072116494
[2, 6]	0.019810675	0.059506297	0.060216019
[2, 7]	0.016998718	0.050811737	0.051458213
[2, 8]	0.014757821	0.044211315	0.044780305
[2, 9]	0.012979253	0.039086531	0.039593936
[2,10]	0.011541316	0.034997334	0.035457604
[2,11]	0.010304198	0.031582094	0.031991352
[2,12]	0.009132843	0.028576344	0.028906167
[2,13]	0.007942064	0.025820013	0.026024717
[2,14]	0.006993196	0.023226828	0.023264244
[2,15]	0.008964656	0.020845503	0.020698531
[3, 3]	0.041058888	0.130854532	0.131516580
[3, 4]	0.021264225	0.098426701	0.099724610
[3, 5]	0.017096441	0.079096691	0.080540853
[3, 6]	0.014578936	0.065983741	0.067349999
[3, 7]	0.012593695	0.056404217	0.057617391
[3, 8]	0.011007575	0.049120440	0.050185204
[3, 9]	0.009742689	0.043458296	0.044406395
[3,10]	0.008707642	0.038934017	0.039790863
[3,11]	0.007799874	0.035149407	0.035916261
[3,12]	0.006924270	0.031814224	0.032462475
[3,13]	0.006026798	0.028753279	0.029233856
[3,14]	0.005334741	0.025872186	0.026139557
[4, 4]	0.023197726	0.108684703	0.110853843
[4, 5]	0.019059353	0.087489764	0.089674639
[4, 6]	0.016305885	0.073067873	0.075067231
[4, 7]	0.014113657	0.062512709	0.064270659
[4, 8]	0.012359299	0.054478041	0.056017612
[4, 9]	0.010958272	0.048226382	0.049595117
[4,10]	0.009808130	0.043225399	0.044459806
[4,11]	0.008794060	0.039036724	0.040143224
[4,12]	0.007810705	0.035341711	0.036291271
[4,13]	0.006800130	0.031948372	0.032688100
[5, 5]	0.021847000	0.096357205	0.099168762
[5, 6]	0.018710940	0.080560049	0.083093763
[5, 7]	0.016199698	0.068978664	0.071194700
[5, 8]	0.014188971	0.060153826	0.062090930
[5, 9]	0.012582765	0.053281373	0.055000954
[5,10]	0.011263682	0.047777717	0.049326095
[5,11]	0.010100041	0.043162291	0.044550211
[5,12]	0.008971080	0.039086788	0.040284154
[6, 6]	0.021048625	0.088655332	0.091609209
[6, 7]	0.018228383	0.075978296	0.078554065
[6, 8]	0.015969289	0.066307447	0.068556166
[6, 9]	0.014163887	0.058768769	0.060762962
[6,10]	0.012680361	0.052724363	0.054518112
[6,11]	0.011370912	0.047648810	0.049255745
[7, 7]	0.020298830	0.083838653	0.086717458
[7, 8]	0.017792800	0.073235934	0.075748662
[7, 9]	0.015788145	0.064959742	0.067187867
[7,10]	0.014138561	0.058313756	0.060317237
[8, 8]	0.019841620	0.081432447	0.084222921
[8, 9]	0.017620480	0.072305381	0.074782259

APPENDIX 5

THE PERFORMANCE OF VARIOUS ESTIMATORS ON THE SET OF NEAR-NORMAL DISTRIBUTIONS, AND THE FACTORS BY WHICH THESE DISTRIBUTIONS ARE SCALED IN ORDER TO STANDARDIZE THEIR 95TH PERCENTILES

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0000000 2: 0.0000000 3: 0.0000000 4: 0.5000000
5: 0.5000000 6: 0.0000000 7: 0.0000000 8: 0.0000000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO:(0, 0, 0, 1).

DISTRI-BUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.089888	97.4	0.074805
(1, 2)	0.75, 1.75	0.0206	0.1232	0.150729	81.7	0.141728
(2, 1)	1.50, 2.50	0.0431	0.1110	0.145608	76.2	0.203807
(2, 2)	1.50, 2.50	0.0210	0.1204	0.155934	77.2	0.185132
(3, 1)	0.25, 2.00	0.2841	0.0145	0.015862	91.4	0.011926
(3, 2)	0.25, 2.00	0.0206	0.1224	0.147043	83.3	0.145884
(3, 3)	0.25, 2.00	0.0164	0.1236	0.151980	81.3	0.150316
(4, 1)	0.75, 2.50	0.1897	0.0387	0.043107	89.7	0.130297
(4, 2)	0.75, 2.50	0.1610	0.0449	0.049781	90.2	0.103409
(4, 3)	0.75, 2.50	0.0869	0.0764	0.084050	91.0	0.119752
(4, 4)	0.75, 2.50	0.0543	0.1015	0.114791	88.4	0.134034
(4, 5)	0.75, 2.50	0.0543	0.1027	0.115903	88.6	0.132830
(4, 6)	0.75, 2.50	0.0408	0.1098	0.126801	86.6	0.142305
(4, 7)	0.75, 2.50	0.0247	0.1189	0.142664	83.3	0.151072
(4, 8)	0.75, 2.50	-.0132	0.1231	0.181756	67.7	0.178644
(5, 1)	1.25, 3.00	0.0965	0.0745	0.102859	72.4	0.222833
(5, 2)	1.25, 3.00	0.0203	0.1185	0.153918	77.0	0.176201
(6, 1)	0.25, 2.25	0.3256	0.0069	0.007704	90.2	0.007127
(6, 2)	0.25, 2.25	0.1648	0.0314	0.033051	95.0	0.031785
(6, 3)	0.25, 2.25	0.0842	0.0808	0.081690	98.9	0.081840
(6, 4)	0.25, 2.25	0.0667	0.0953	0.098227	97.1	0.098641
(6, 5)	0.25, 2.25	0.0371	0.1162	0.129625	89.6	0.128461
(6, 6)	0.25, 2.25	0.0198	0.1232	0.149953	82.1	0.148603
(7, 1)	0.50, 2.50	0.2673	0.0161	0.017365	92.6	0.063446
(7, 2)	0.50, 2.50	0.2673	0.0282	0.030963	91.1	0.039978
(7, 3)	0.50, 2.50	0.1570	0.0430	0.044747	96.2	0.069760
(7, 4)	0.50, 2.50	0.1570	0.0441	0.045904	96.2	0.067612
(7, 5)	0.50, 2.50	0.0924	0.0770	0.079485	96.9	0.098299
(7, 6)	0.50, 2.50	0.0368	0.1126	0.126464	89.0	0.131370
(7, 7)	0.50, 2.50	0.0208	0.1212	0.145005	83.6	0.146973
(7, 8)	0.50, 2.50	-.0123	0.1235	0.180563	68.4	0.177296
(7, 9)	0.50, 2.50	-.0216	0.1208	0.191148	63.2	0.189534
(8, 1)	0.75, 2.75	0.2055	0.0319	0.037263	85.6	0.204048
(8, 2)	0.75, 2.75	0.1679	0.0420	0.048571	86.5	0.184525
(8, 3)	0.75, 2.75	0.1091	0.0617	0.069052	89.4	0.136956
(8, 4)	0.75, 2.75	0.0795	0.0785	0.087647	89.6	0.129437
(8, 5)	0.75, 2.75	0.0410	0.1039	0.118104	88.0	0.142463
(8, 6)	0.75, 2.75	0.0172	0.1206	0.148986	81.0	0.158335
(9, 1)	1.00, 3.00	0.1484	0.0538	0.066927	80.4	0.217993
(9, 2)	1.00, 3.00	0.0861	0.0763	0.093946	81.2	0.188225
(9, 3)	1.00, 3.00	0.0473	0.1020	0.125495	81.3	0.162953
(9, 4)	1.00, 3.00	0.0263	0.1141	0.146297	78.0	0.171317
(10, 1)	0.25, 2.50	0.3556	0.0041	0.004478	92.0	0.012040
(10, 2)	0.25, 2.50	0.0193	0.1232	0.150150	82.1	0.151041
(11, 1)	0.75, 3.00	0.2153	0.0292	0.034804	83.8	0.205819
(11, 2)	0.75, 3.00	0.0193	0.1188	0.145728	81.5	0.157208
(11, 3)	0.75, 3.00	0.0151	0.1209	0.150546	80.3	0.160462
(12, 1)	0.00, 0.00	0.0000	0.1250	0.169144	73.9	0.172146

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0000000 2: 0.0000000 3: 0.1666667 4: 0.3333333
5: 0.3333333 6: 0.1666667 7: 0.0000000 8: 0.0000000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO: (0, 0, 1, 2).

DISTRI-BUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.087792	99.7	0.073061
(1, 2)	0.75, 1.75	0.0206	0.1232	0.138319	89.1	0.130059
(2, 1)	1.50, 2.50	0.0431	0.1110	0.130088	85.3	0.182084
(2, 2)	1.50, 2.50	0.0210	0.1204	0.140502	85.7	0.166810
(3, 1)	0.25, 2.00	0.2841	0.0145	0.022228	65.2	0.016713
(3, 2)	0.25, 2.00	0.0206	0.1224	0.135891	90.1	0.134820
(3, 3)	0.25, 2.00	0.0164	0.1236	0.139800	88.4	0.138268
(4, 1)	0.75, 2.50	0.1897	0.0387	0.040227	96.2	0.121592
(4, 2)	0.75, 2.50	0.1610	0.0449	0.046475	96.6	0.096541
(4, 3)	0.75, 2.50	0.0869	0.0764	0.078288	97.7	0.111544
(4, 4)	0.75, 2.50	0.0543	0.1015	0.105824	95.9	0.123564
(4, 5)	0.75, 2.50	0.0543	0.1027	0.106901	96.1	0.122513
(4, 6)	0.75, 2.50	0.0408	0.1098	0.116496	94.3	0.130740
(4, 7)	0.75, 2.50	0.0247	0.1189	0.130442	91.1	0.138130
(4, 8)	0.75, 2.50	-.0132	0.1231	0.163216	75.4	0.160421
(5, 1)	1.25, 3.00	0.0965	0.0745	0.091402	81.5	0.198014
(5, 2)	1.25, 3.00	0.0203	0.1185	0.138372	85.7	0.158404
(6, 1)	0.25, 2.25	0.3256	0.0069	0.011042	62.9	0.010215
(6, 2)	0.25, 2.25	0.1648	0.0314	0.037526	83.7	0.036088
(6, 3)	0.25, 2.25	0.0842	0.0808	0.081575	99.0	0.081725
(6, 4)	0.25, 2.25	0.0667	0.0953	0.095814	99.5	0.096219
(6, 5)	0.25, 2.25	0.0371	0.1162	0.122119	95.1	0.121023
(6, 6)	0.25, 2.25	0.0198	0.1232	0.138210	89.1	0.136966
(7, 1)	0.50, 2.50	0.2673	0.0161	0.017224	93.3	0.062932
(7, 2)	0.50, 2.50	0.2673	0.0282	0.029804	94.6	0.038481
(7, 3)	0.50, 2.50	0.1570	0.0430	0.045057	95.5	0.070242
(7, 4)	0.50, 2.50	0.1570	0.0441	0.046431	95.1	0.068390
(7, 5)	0.50, 2.50	0.0924	0.0770	0.077258	99.7	0.095544
(7, 6)	0.50, 2.50	0.0368	0.1126	0.117794	95.6	0.122363
(7, 7)	0.50, 2.50	0.0208	0.1212	0.133502	90.8	0.135313
(7, 8)	0.50, 2.50	-.0123	0.1235	0.162045	76.2	0.159113
(7, 9)	0.50, 2.50	-.0216	0.1208	0.170185	71.0	0.168748
(8, 1)	0.75, 2.75	0.2055	0.0319	0.033867	94.1	0.185455
(8, 2)	0.75, 2.75	0.1679	0.0420	0.044343	94.7	0.168462
(8, 3)	0.75, 2.75	0.1091	0.0617	0.063955	96.5	0.126849
(8, 4)	0.75, 2.75	0.0795	0.0785	0.081256	96.6	0.119999
(8, 5)	0.75, 2.75	0.0410	0.1039	0.109359	95.0	0.131914
(8, 6)	0.75, 2.75	0.0172	0.1206	0.135689	88.9	0.144203
(9, 1)	1.00, 3.00	0.1484	0.0538	0.059848	89.9	0.194935
(9, 2)	1.00, 3.00	0.0861	0.0763	0.084612	90.1	0.169524
(9, 3)	1.00, 3.00	0.0473	0.1020	0.113362	90.0	0.147198
(9, 4)	1.00, 3.00	0.0263	0.1141	0.131668	86.6	0.154185
(10, 1)	0.25, 2.50	0.3556	0.0041	0.005770	71.4	0.015513
(10, 2)	0.25, 2.50	0.0193	0.1232	0.138583	88.9	0.139406
(11, 1)	0.75, 3.00	0.2153	0.0292	0.031391	92.9	0.185639
(11, 2)	0.75, 3.00	0.0193	0.1188	0.132796	89.5	0.143258
(11, 3)	0.75, 3.00	0.0151	0.1209	0.136987	88.3	0.146009
(12, 1)	0.00, 0.00	0.0000	0.1250	0.152970	81.7	0.155684

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0000000 2: 0.0000000 3: 0.2500000 4: 0.2500000
5: 0.2500000 6: 0.2500000 7: 0.0000000 8: 0.0000000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO: (0, 0, 1, 1).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.089372	98.0	0.074376
(1, 2)	0.75, 1.75	0.0206	0.1232	0.135527	90.9	0.127434
(2, 1)	1.50, 2.50	0.0431	0.1110	0.125463	88.5	0.175612
(2, 2)	1.50, 2.50	0.0210	0.1204	0.136178	88.4	0.161675
(3, 1)	0.25, 2.00	0.2841	0.0145	0.027915	51.9	0.020989
(3, 2)	0.25, 2.00	0.0206	0.1224	0.133688	91.6	0.132634
(3, 3)	0.25, 2.00	0.0164	0.1236	0.137154	90.1	0.135652
(4, 1)	0.75, 2.50	0.1897	0.0387	0.040007	96.7	0.120925
(4, 2)	0.75, 2.50	0.1610	0.0449	0.046151	97.3	0.095867
(4, 3)	0.75, 2.50	0.0869	0.0764	0.077436	98.7	0.110329
(4, 4)	0.75, 2.50	0.0543	0.1015	0.103991	97.6	0.121423
(4, 5)	0.75, 2.50	0.0543	0.1027	0.105081	97.8	0.120428
(4, 6)	0.75, 2.50	0.0408	0.1098	0.114231	96.1	0.128198
(4, 7)	0.75, 2.50	0.0247	0.1189	0.127533	93.2	0.135049
(4, 8)	0.75, 2.50	-.0132	0.1231	0.157874	78.0	0.155170
(5, 1)	1.25, 3.00	0.0965	0.0745	0.087868	84.8	0.190357
(5, 2)	1.25, 3.00	0.0203	0.1185	0.133937	88.5	0.153327
(6, 1)	0.25, 2.25	0.3256	0.0069	0.014180	49.0	0.013118
(6, 2)	0.25, 2.25	0.1648	0.0314	0.041616	75.5	0.040021
(6, 3)	0.25, 2.25	0.0842	0.0808	0.084031	96.1	0.084185
(6, 4)	0.25, 2.25	0.0667	0.0953	0.097339	97.9	0.097750
(6, 5)	0.25, 2.25	0.0371	0.1162	0.121518	95.6	0.120428
(6, 6)	0.25, 2.25	0.0198	0.1232	0.135757	90.7	0.134535
(7, 1)	0.50, 2.50	0.2673	0.0161	0.017912	89.8	0.065443
(7, 2)	0.50, 2.50	0.2673	0.0282	0.030574	92.2	0.039475
(7, 3)	0.50, 2.50	0.1570	0.0430	0.046722	92.1	0.072838
(7, 4)	0.50, 2.50	0.1570	0.0441	0.048287	91.4	0.071123
(7, 5)	0.50, 2.50	0.0924	0.0770	0.078322	98.3	0.096860
(7, 6)	0.50, 2.50	0.0368	0.1126	0.116427	96.7	0.120944
(7, 7)	0.50, 2.50	0.0208	0.1212	0.131045	92.5	0.132823
(7, 8)	0.50, 2.50	-.0123	0.1235	0.156689	78.8	0.153854
(7, 9)	0.50, 2.50	-.0216	0.1208	0.163791	73.8	0.162408
(8, 1)	0.75, 2.75	0.2055	0.0319	0.033080	96.4	0.181144
(8, 2)	0.75, 2.75	0.1679	0.0420	0.043401	96.7	0.164884
(8, 3)	0.75, 2.75	0.1091	0.0617	0.063048	97.9	0.125048
(8, 4)	0.75, 2.75	0.0795	0.0785	0.080118	98.0	0.118317
(8, 5)	0.75, 2.75	0.0410	0.1039	0.107718	96.5	0.129934
(8, 6)	0.75, 2.75	0.0172	0.1206	0.132347	91.1	0.140652
(9, 1)	1.00, 3.00	0.1484	0.0538	0.057795	93.1	0.188249
(9, 2)	1.00, 3.00	0.0861	0.0763	0.082009	93.0	0.164309
(9, 3)	1.00, 3.00	0.0473	0.1020	0.110051	92.7	0.142899
(9, 4)	1.00, 3.00	0.0263	0.1141	0.127534	89.4	0.149344
(10, 1)	0.25, 2.50	0.3556	0.0041	0.007142	57.7	0.019203
(10, 2)	0.25, 2.50	0.0193	0.1232	0.136231	90.5	0.137039
(11, 1)	0.75, 3.00	0.2153	0.0292	0.030501	95.6	0.180375
(11, 2)	0.75, 3.00	0.0193	0.1188	0.129570	91.7	0.139778
(11, 3)	0.75, 3.00	0.0151	0.1209	0.133541	90.6	0.142336
(12, 1)	0.00, 0.00	0.0000	0.1250	0.148580	84.1	0.151217

THE COEFFICIENTS OF THE ESTIMATOR ARE:

1:	0.0000000	2:	0.0000000	3:	0.3333333	4:	0.1666667
5:	0.1666667	6:	0.3333333	7:	0.0000000	8:	0.0000000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO: (0, 0, 2, 1).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.092704	94.5	0.077149
(1, 2)	0.75, 1.75	0.0206	0.1232	0.135010	91.3	0.126948
(2, 1)	1.50, 2.50	0.0431	0.1110	0.122930	90.3	0.172065
(2, 2)	1.50, 2.50	0.0210	0.1204	0.134113	89.8	0.159225
(3, 1)	0.25, 2.00	0.2841	0.0145	0.035272	41.1	0.026520
(3, 2)	0.25, 2.00	0.0206	0.1224	0.133734	91.6	0.132679
(3, 3)	0.25, 2.00	0.0164	0.1236	0.136805	90.3	0.135307
(4, 1)	0.75, 2.50	0.1897	0.0387	0.040599	95.3	0.122716
(4, 2)	0.75, 2.50	0.1610	0.0449	0.046712	96.1	0.097033
(4, 3)	0.75, 2.50	0.0869	0.0764	0.077935	98.1	0.111040
(4, 4)	0.75, 2.50	0.0543	0.1015	0.103923	97.6	0.121345
(4, 5)	0.75, 2.50	0.0543	0.1027	0.105049	97.8	0.120391
(4, 6)	0.75, 2.50	0.0408	0.1098	0.113890	96.4	0.127816
(4, 7)	0.75, 2.50	0.0247	0.1189	0.126757	93.8	0.134227
(4, 8)	0.75, 2.50	-.0132	0.1231	0.155149	79.4	0.152492
(5, 1)	1.25, 3.00	0.0965	0.0745	0.085795	86.8	0.185866
(5, 2)	1.25, 3.00	0.0203	0.1185	0.131728	90.0	0.150798
(6, 1)	0.25, 2.25	0.3256	0.0069	0.018296	38.0	0.016926
(6, 2)	0.25, 2.25	0.1648	0.0314	0.046941	66.9	0.045142
(6, 3)	0.25, 2.25	0.0842	0.0808	0.088163	91.6	0.088325
(6, 4)	0.25, 2.25	0.0667	0.0953	0.100685	94.7	0.101110
(6, 5)	0.25, 2.25	0.0371	0.1162	0.123018	94.4	0.121914
(6, 6)	0.25, 2.25	0.0198	0.1232	0.135582	90.8	0.134362
(7, 1)	0.50, 2.50	0.2673	0.0161	0.019104	84.2	0.069799
(7, 2)	0.50, 2.50	0.2673	0.0282	0.032244	87.5	0.041632
(7, 3)	0.50, 2.50	0.1570	0.0430	0.049393	87.1	0.077003
(7, 4)	0.50, 2.50	0.1570	0.0441	0.051204	86.2	0.075419
(7, 5)	0.50, 2.50	0.0924	0.0770	0.080837	95.2	0.099970
(7, 6)	0.50, 2.50	0.0368	0.1126	0.117040	96.2	0.121580
(7, 7)	0.50, 2.50	0.0208	0.1212	0.130785	92.7	0.132560
(7, 8)	0.50, 2.50	-.0123	0.1235	0.153935	80.3	0.151150
(7, 9)	0.50, 2.50	-.0216	0.1208	0.160122	75.5	0.158769
(8, 1)	0.75, 2.75	0.2055	0.0319	0.032900	96.9	0.180157
(8, 2)	0.75, 2.75	0.1679	0.0420	0.043241	97.1	0.164276
(8, 3)	0.75, 2.75	0.1091	0.0617	0.063233	97.6	0.125416
(8, 4)	0.75, 2.75	0.0795	0.0785	0.080351	97.7	0.118662
(8, 5)	0.75, 2.75	0.0410	0.1039	0.107896	96.3	0.130149
(8, 6)	0.75, 2.75	0.0172	0.1206	0.131210	91.9	0.139444
(9, 1)	1.00, 3.00	0.1484	0.0538	0.056734	94.9	0.184792
(9, 2)	1.00, 3.00	0.0861	0.0763	0.080782	94.4	0.161851
(9, 3)	1.00, 3.00	0.0473	0.1020	0.108577	93.9	0.140985
(9, 4)	1.00, 3.00	0.0263	0.1141	0.125521	90.9	0.146987
(10, 1)	0.25, 2.50	0.3556	0.0041	0.008999	45.8	0.024196
(10, 2)	0.25, 2.50	0.0193	0.1232	0.136166	90.5	0.136974
(11, 1)	0.75, 3.00	0.2153	0.0292	0.030155	96.7	0.178328
(11, 2)	0.75, 3.00	0.0193	0.1188	0.128503	92.5	0.138627
(11, 3)	0.75, 3.00	0.0151	0.1209	0.132318	91.4	0.141033
(12, 1)	0.00, 0.00	0.0000	0.1250	0.146657	85.2	0.149259

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0000000 2: 0.1250000 3: 0.1250000 4: 0.2500000
5: 0.2500000 6: 0.1250000 7: 0.1250000 8: 0.0000000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO: (0, 1, 1, 2).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.094875	92.3	0.078955
(1, 2)	0.75, 1.75	0.0206	0.1232	0.129349	95.2	0.121625
(2, 1)	1.50, 2.50	0.0431	0.1110	0.116665	95.2	0.163296
(2, 2)	1.50, 2.50	0.0210	0.1204	0.126875	94.9	0.150631
(3, 1)	0.25, 2.00	0.2841	0.0145	0.044006	33.0	0.033087
(3, 2)	0.25, 2.00	0.0206	0.1224	0.127330	96.2	0.126326
(3, 3)	0.25, 2.00	0.0164	0.1236	0.130007	95.0	0.128583
(4, 1)	0.75, 2.50	0.1897	0.0387	0.046095	83.9	0.139327
(4, 2)	0.75, 2.50	0.1610	0.0449	0.051301	87.5	0.106566
(4, 3)	0.75, 2.50	0.0869	0.0764	0.079030	96.7	0.112600
(4, 4)	0.75, 2.50	0.0543	0.1015	0.101910	99.6	0.118994
(4, 5)	0.75, 2.50	0.0543	0.1027	0.103184	99.6	0.118254
(4, 6)	0.75, 2.50	0.0408	0.1098	0.110437	99.4	0.123941
(4, 7)	0.75, 2.50	0.0247	0.1189	0.121528	97.8	0.128690
(4, 8)	0.75, 2.50	-.0132	0.1231	0.145658	84.5	0.143164
(5, 1)	1.25, 3.00	0.0965	0.0745	0.080399	92.7	0.174176
(5, 2)	1.25, 3.00	0.0203	0.1185	0.124788	95.0	0.142853
(6, 1)	0.25, 2.25	0.3256	0.0069	0.028132	24.7	0.026025
(6, 2)	0.25, 2.25	0.1648	0.0314	0.053849	58.3	0.051786
(6, 3)	0.25, 2.25	0.0842	0.0808	0.088507	91.3	0.088669
(6, 4)	0.25, 2.25	0.0667	0.0953	0.099116	96.2	0.099534
(6, 5)	0.25, 2.25	0.0371	0.1162	0.118095	98.4	0.117036
(6, 6)	0.25, 2.25	0.0198	0.1232	0.128996	95.5	0.127834
(7, 1)	0.50, 2.50	0.2673	0.0161	0.026005	61.8	0.095014
(7, 2)	0.50, 2.50	0.2673	0.0282	0.041828	67.4	0.054006
(7, 3)	0.50, 2.50	0.1570	0.0430	0.055584	77.4	0.086655
(7, 4)	0.50, 2.50	0.1570	0.0441	0.058070	76.0	0.085532
(7, 5)	0.50, 2.50	0.0924	0.0770	0.082561	93.3	0.102102
(7, 6)	0.50, 2.50	0.0368	0.1126	0.113206	99.5	0.117597
(7, 7)	0.50, 2.50	0.0208	0.1212	0.124866	97.1	0.126560
(7, 8)	0.50, 2.50	-.0123	0.1235	0.144816	85.3	0.142196
(7, 9)	0.50, 2.50	-.0216	0.1208	0.150061	80.5	0.148794
(8, 1)	0.75, 2.75	0.2055	0.0319	0.035614	89.5	0.195020
(8, 2)	0.75, 2.75	0.1679	0.0420	0.045963	91.4	0.174615
(8, 3)	0.75, 2.75	0.1091	0.0617	0.064311	96.0	0.127554
(8, 4)	0.75, 2.75	0.0795	0.0785	0.079881	98.3	0.117968
(8, 5)	0.75, 2.75	0.0410	0.1039	0.104145	99.8	0.125625
(8, 6)	0.75, 2.75	0.0172	0.1206	0.124855	96.6	0.132690
(9, 1)	1.00, 3.00	0.1484	0.0538	0.056366	95.5	0.183595
(9, 2)	1.00, 3.00	0.0861	0.0763	0.078543	97.1	0.157365
(9, 3)	1.00, 3.00	0.0473	0.1020	0.104287	97.8	0.135414
(9, 4)	1.00, 3.00	0.0263	0.1141	0.119069	95.8	0.139431
(10, 1)	0.25, 2.50	0.3556	0.0041	0.017114	24.1	0.046014
(10, 2)	0.25, 2.50	0.0193	0.1232	0.129168	95.4	0.129934
(11, 1)	0.75, 3.00	0.2153	0.0292	0.031931	91.3	0.188828
(11, 2)	0.75, 3.00	0.0193	0.1188	0.122362	97.1	0.132001
(11, 3)	0.75, 3.00	0.0151	0.1209	0.125694	96.2	0.133973
(12, 1)	0.00, 0.00	0.0000	0.1250	0.138300	90.4	0.140754

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0000000 2: 0.1000000 3: 0.2000000 4: 0.2000000
5: 0.2000000 6: 0.2000000 7: 0.1000000 8: 0.0000000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO:(0, 1, 2, 2).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.094830	92.3	0.078918
(1, 2)	0.75, 1.75	0.0206	0.1232	0.129319	95.3	0.121596
(2, 1)	1.50, 2.50	0.0431	0.1110	0.116074	95.6	0.162469
(2, 2)	1.50, 2.50	0.0210	0.1204	0.126603	95.1	0.150308
(3, 1)	0.25, 2.00	0.2841	0.0145	0.044017	32.9	0.033096
(3, 2)	0.25, 2.00	0.0206	0.1224	0.127671	95.9	0.126664
(3, 3)	0.25, 2.00	0.0164	0.1236	0.130264	94.9	0.128837
(4, 1)	0.75, 2.50	0.1897	0.0387	0.044507	86.9	0.134530
(4, 2)	0.75, 2.50	0.1610	0.0449	0.049934	89.9	0.103727
(4, 3)	0.75, 2.50	0.0869	0.0764	0.078297	97.6	0.111555
(4, 4)	0.75, 2.50	0.0543	0.1015	0.101484	100.0	0.118497
(4, 5)	0.75, 2.50	0.0543	0.1027	0.102730	100.0	0.117733
(4, 6)	0.75, 2.50	0.0408	0.1098	0.110151	99.7	0.123620
(4, 7)	0.75, 2.50	0.0247	0.1189	0.121369	98.0	0.128521
(4, 8)	0.75, 2.50	-.0132	0.1231	0.145471	84.6	0.142980
(5, 1)	1.25, 3.00	0.0965	0.0745	0.080041	93.1	0.173401
(5, 2)	1.25, 3.00	0.0203	0.1185	0.124422	95.3	0.142434
(6, 1)	0.25, 2.25	0.3256	0.0069	0.026709	26.0	0.024708
(6, 2)	0.25, 2.25	0.1648	0.0314	0.053816	58.4	0.051754
(6, 3)	0.25, 2.25	0.0842	0.0808	0.089233	90.5	0.089397
(6, 4)	0.25, 2.25	0.0667	0.0953	0.099875	95.5	0.100297
(6, 5)	0.25, 2.25	0.0371	0.1162	0.118733	97.8	0.117667
(6, 6)	0.25, 2.25	0.0198	0.1232	0.129280	95.3	0.128116
(7, 1)	0.50, 2.50	0.2673	0.0161	0.024388	65.9	0.089105
(7, 2)	0.50, 2.50	0.2673	0.0282	0.039503	71.4	0.051004
(7, 3)	0.50, 2.50	0.1570	0.0430	0.054690	78.7	0.085261
(7, 4)	0.50, 2.50	0.1570	0.0441	0.057086	77.3	0.084083
(7, 5)	0.50, 2.50	0.0924	0.0770	0.082435	93.4	0.101947
(7, 6)	0.50, 2.50	0.0368	0.1126	0.113332	99.4	0.117728
(7, 7)	0.50, 2.50	0.0208	0.1212	0.125015	97.0	0.126711
(7, 8)	0.50, 2.50	-.0123	0.1235	0.144550	85.5	0.141935
(7, 9)	0.50, 2.50	-.0216	0.1208	0.149609	80.8	0.148345
(8, 1)	0.75, 2.75	0.2055	0.0319	0.034452	92.5	0.188659
(8, 2)	0.75, 2.75	0.1679	0.0420	0.044780	93.8	0.170122
(8, 3)	0.75, 2.75	0.1091	0.0617	0.063560	97.1	0.126064
(8, 4)	0.75, 2.75	0.0795	0.0785	0.079388	98.9	0.117240
(8, 5)	0.75, 2.75	0.0410	0.1039	0.104152	99.8	0.125632
(8, 6)	0.75, 2.75	0.0172	0.1206	0.124769	96.7	0.132598
(9, 1)	1.00, 3.00	0.1484	0.0538	0.055407	97.1	0.180471
(9, 2)	1.00, 3.00	0.0861	0.0763	0.077863	97.9	0.156001
(9, 3)	1.00, 3.00	0.0473	0.1020	0.103786	98.3	0.134764
(9, 4)	1.00, 3.00	0.0263	0.1141	0.118718	96.1	0.139021
(10, 1)	0.25, 2.50	0.3556	0.0041	0.015325	26.9	0.041202
(10, 2)	0.25, 2.50	0.0193	0.1232	0.129574	95.1	0.130343
(11, 1)	0.75, 3.00	0.2153	0.0292	0.030932	94.3	0.182919
(11, 2)	0.75, 3.00	0.0193	0.1188	0.122277	97.2	0.131910
(11, 3)	0.75, 3.00	0.0151	0.1209	0.125626	96.3	0.133900
(12, 1)	0.00, 0.00	0.0000	0.1250	0.138233	90.4	0.140686

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0000000 2: 0.1666667 3: 0.1666667 4: 0.1666667
5: 0.1666667 6: 0.1666667 7: 0.1666667 8: 0.0000000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO:(0, 1, 1, 1).

DISTRI-BUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.102124	85.7	0.084988
(1, 2)	0.75, 1.75	0.0206	0.1232	0.128239	96.1	0.120582
(2, 1)	1.50, 2.50	0.0431	0.1110	0.112550	98.6	0.157537
(2, 2)	1.50, 2.50	0.0210	0.1204	0.123063	97.8	0.146105
(3, 1)	0.25, 2.00	0.2841	0.0145	0.061594	23.5	0.046311
(3, 2)	0.25, 2.00	0.0206	0.1224	0.126659	96.7	0.125660
(3, 3)	0.25, 2.00	0.0164	0.1236	0.128676	96.0	0.127266
(4, 1)	0.75, 2.50	0.1897	0.0387	0.050871	76.0	0.153766
(4, 2)	0.75, 2.50	0.1610	0.0449	0.055527	80.9	0.115344
(4, 3)	0.75, 2.50	0.0869	0.0764	0.081683	93.6	0.116380
(4, 4)	0.75, 2.50	0.0543	0.1015	0.102627	98.9	0.119831
(4, 5)	0.75, 2.50	0.0543	0.1027	0.104037	98.7	0.119231
(4, 6)	0.75, 2.50	0.0408	0.1098	0.110269	99.6	0.123752
(4, 7)	0.75, 2.50	0.0247	0.1189	0.120162	98.9	0.127243
(4, 8)	0.75, 2.50	-.0132	0.1231	0.140277	87.8	0.137875
(5, 1)	1.25, 3.00	0.0965	0.0745	0.076833	97.0	0.166450
(5, 2)	1.25, 3.00	0.0203	0.1185	0.120860	98.1	0.138357
(6, 1)	0.25, 2.25	0.3256	0.0069	0.041363	16.8	0.038265
(6, 2)	0.25, 2.25	0.1648	0.0314	0.066408	47.3	0.063863
(6, 3)	0.25, 2.25	0.0842	0.0808	0.096031	84.1	0.096207
(6, 4)	0.25, 2.25	0.0667	0.0953	0.104746	91.0	0.105188
(6, 5)	0.25, 2.25	0.0371	0.1162	0.119912	96.9	0.118836
(6, 6)	0.25, 2.25	0.0198	0.1232	0.127979	96.2	0.126827
(7, 1)	0.50, 2.50	0.2673	0.0161	0.032360	49.7	0.118230
(7, 2)	0.50, 2.50	0.2673	0.0282	0.050610	55.7	0.065345
(7, 3)	0.50, 2.50	0.1570	0.0430	0.063634	67.6	0.099204
(7, 4)	0.50, 2.50	0.1570	0.0441	0.066891	66.0	0.098525
(7, 5)	0.50, 2.50	0.0924	0.0770	0.088357	87.1	0.109270
(7, 6)	0.50, 2.50	0.0368	0.1126	0.114209	98.6	0.118639
(7, 7)	0.50, 2.50	0.0208	0.1212	0.123948	97.8	0.125630
(7, 8)	0.50, 2.50	-.0123	0.1235	0.139529	88.5	0.137005
(7, 9)	0.50, 2.50	-.0216	0.1208	0.143268	84.3	0.142059
(8, 1)	0.75, 2.75	0.2055	0.0319	0.037768	84.4	0.206815
(8, 2)	0.75, 2.75	0.1679	0.0420	0.048206	87.1	0.183138
(8, 3)	0.75, 2.75	0.1091	0.0617	0.066344	93.0	0.131586
(8, 4)	0.75, 2.75	0.0795	0.0785	0.081432	96.4	0.120258
(8, 5)	0.75, 2.75	0.0410	0.1039	0.104487	99.5	0.126037
(8, 6)	0.75, 2.75	0.0172	0.1206	0.122581	98.4	0.130273
(9, 1)	1.00, 3.00	0.1484	0.0538	0.056174	95.8	0.182970
(9, 2)	1.00, 3.00	0.0861	0.0763	0.077385	98.5	0.155044
(9, 3)	1.00, 3.00	0.0473	0.1020	0.102191	99.8	0.132692
(9, 4)	1.00, 3.00	0.0263	0.1141	0.115517	98.7	0.135272
(10, 1)	0.25, 2.50	0.3556	0.0041	0.025683	16.0	0.069052
(10, 2)	0.25, 2.50	0.0193	0.1232	0.128130	96.2	0.128890
(11, 1)	0.75, 3.00	0.2153	0.0292	0.033370	87.4	0.197339
(11, 2)	0.75, 3.00	0.0193	0.1188	0.120231	98.8	0.129703
(11, 3)	0.75, 3.00	0.0151	0.1209	0.123204	98.2	0.131318
(12, 1)	0.00, 0.00	0.0000	0.1250	0.134337	93.0	0.136721

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0625000 2: 0.0625000 3: 0.1250000 4: 0.2500000
5: 0.2500000 6: 0.1250000 7: 0.0625000 8: 0.0625000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO:(1, 1, 2, 4).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.096853	90.4	0.080601
(1, 2)	0.75, 1.75	0.0206	0.1232	0.125269	98.3	0.117788
(2, 1)	1.50, 2.50	0.0431	0.1110	0.119610	92.8	0.167419
(2, 2)	1.50, 2.50	0.0210	0.1204	0.125879	95.7	0.149449
(3, 1)	0.25, 2.00	0.2841	0.0145	0.053332	27.2	0.040099
(3, 2)	0.25, 2.00	0.0206	0.1224	0.123806	98.9	0.122830
(3, 3)	0.25, 2.00	0.0164	0.1236	0.125951	98.1	0.124572
(4, 1)	0.75, 2.50	0.1897	0.0387	0.065164	59.4	0.196967
(4, 2)	0.75, 2.50	0.1610	0.0449	0.069361	64.7	0.144082
(4, 3)	0.75, 2.50	0.0869	0.0764	0.089892	85.0	0.128075
(4, 4)	0.75, 2.50	0.0543	0.1015	0.106498	95.3	0.124351
(4, 5)	0.75, 2.50	0.0543	0.1027	0.107258	95.8	0.122922
(4, 6)	0.75, 2.50	0.0408	0.1098	0.112689	97.5	0.126468
(4, 7)	0.75, 2.50	0.0247	0.1189	0.120647	98.5	0.127757
(4, 8)	0.75, 2.50	-.0132	0.1231	0.138502	88.9	0.136130
(5, 1)	1.25, 3.00	0.0965	0.0745	0.094167	79.1	0.204003
(5, 2)	1.25, 3.00	0.0203	0.1185	0.124537	95.2	0.142566
(6, 1)	0.25, 2.25	0.3256	0.0069	0.044326	15.7	0.041006
(6, 2)	0.25, 2.25	0.1648	0.0314	0.064939	48.4	0.062451
(6, 3)	0.25, 2.25	0.0842	0.0808	0.092824	87.0	0.092994
(6, 4)	0.25, 2.25	0.0667	0.0953	0.101226	94.2	0.101653
(6, 5)	0.25, 2.25	0.0371	0.1162	0.116216	100.0	0.115172
(6, 6)	0.25, 2.25	0.0198	0.1232	0.125108	98.4	0.123982
(7, 1)	0.50, 2.50	0.2673	0.0161	0.048723	33.0	0.178018
(7, 2)	0.50, 2.50	0.2673	0.0282	0.058621	48.1	0.075688
(7, 3)	0.50, 2.50	0.1570	0.0430	0.069576	61.8	0.108467
(7, 4)	0.50, 2.50	0.1570	0.0441	0.070731	62.4	0.104181
(7, 5)	0.50, 2.50	0.0924	0.0770	0.090025	85.5	0.111333
(7, 6)	0.50, 2.50	0.0368	0.1126	0.113662	99.1	0.118071
(7, 7)	0.50, 2.50	0.0208	0.1212	0.122432	99.0	0.124093
(7, 8)	0.50, 2.50	-.0123	0.1235	0.137899	89.6	0.135404
(7, 9)	0.50, 2.50	-.0216	0.1208	0.142178	85.0	0.140978
(8, 1)	0.75, 2.75	0.2055	0.0319	0.059226	53.8	0.324321
(8, 2)	0.75, 2.75	0.1679	0.0420	0.066871	62.8	0.254048
(8, 3)	0.75, 2.75	0.1091	0.0617	0.080063	77.1	0.158796
(8, 4)	0.75, 2.75	0.0795	0.0785	0.091141	86.1	0.134596
(8, 5)	0.75, 2.75	0.0410	0.1039	0.108263	96.0	0.130592
(8, 6)	0.75, 2.75	0.0172	0.1206	0.123400	97.7	0.131144
(9, 1)	1.00, 3.00	0.1484	0.0538	0.075816	71.0	0.246947
(9, 2)	1.00, 3.00	0.0861	0.0763	0.092006	82.9	0.184338
(9, 3)	1.00, 3.00	0.0473	0.1020	0.109980	92.7	0.142806
(9, 4)	1.00, 3.00	0.0263	0.1141	0.120468	94.7	0.141070
(10, 1)	0.25, 2.50	0.3556	0.0041	0.039182	10.5	0.105347
(10, 2)	0.25, 2.50	0.0193	0.1232	0.125253	98.4	0.125996
(11, 1)	0.75, 3.00	0.2153	0.0292	0.056794	51.4	0.335862
(11, 2)	0.75, 3.00	0.0193	0.1188	0.121639	97.7	0.131221
(11, 3)	0.75, 3.00	0.0151	0.1209	0.124054	97.5	0.132224
(12, 1)	0.00, 0.00	0.0000	0.1250	0.133004	94.0	0.135364

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0714286 2: 0.1428571 3: 0.1428571 4: 0.1428571
5: 0.1428571 6: 0.1428571 7: 0.1428571 8: 0.0714286

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO: (1, 2, 2, 2).

DISTRI-BUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.111408	78.6	0.092714
(1, 2)	0.75, 1.75	0.0206	0.1232	0.124086	99.3	0.116676
(2, 1)	1.50, 2.50	0.0431	0.1110	0.115486	96.1	0.161646
(2, 2)	1.50, 2.50	0.0210	0.1204	0.121107	99.4	0.143783
(3, 1)	0.25, 2.00	0.2841	0.0145	0.089573	16.2	0.067348
(3, 2)	0.25, 2.00	0.0206	0.1224	0.123250	99.3	0.122279
(3, 3)	0.25, 2.00	0.0164	0.1236	0.124221	99.5	0.122860
(4, 1)	0.75, 2.50	0.1897	0.0387	0.081970	47.2	0.247765
(4, 2)	0.75, 2.50	0.1610	0.0449	0.084702	53.0	0.175950
(4, 3)	0.75, 2.50	0.0869	0.0764	0.099683	76.7	0.142026
(4, 4)	0.75, 2.50	0.0543	0.1015	0.110926	91.5	0.129521
(4, 5)	0.75, 2.50	0.0543	0.1027	0.111790	91.9	0.128116
(4, 6)	0.75, 2.50	0.0408	0.1098	0.114884	95.6	0.128931
(4, 7)	0.75, 2.50	0.0247	0.1189	0.119926	99.1	0.126994
(4, 8)	0.75, 2.50	-.0132	0.1231	0.129657	95.0	0.127437
(5, 1)	1.25, 3.00	0.0965	0.0745	0.093527	79.7	0.202616
(5, 2)	1.25, 3.00	0.0203	0.1185	0.119830	98.9	0.137177
(6, 1)	0.25, 2.25	0.3256	0.0069	0.076964	9.0	0.071199
(6, 2)	0.25, 2.25	0.1648	0.0314	0.092390	34.0	0.088850
(6, 3)	0.25, 2.25	0.0842	0.0808	0.108124	74.7	0.108322
(6, 4)	0.25, 2.25	0.0667	0.0953	0.112549	84.7	0.113024
(6, 5)	0.25, 2.25	0.0371	0.1162	0.120039	96.8	0.118961
(6, 6)	0.25, 2.25	0.0198	0.1232	0.123921	99.4	0.122805
(7, 1)	0.50, 2.50	0.2673	0.0161	0.070279	22.9	0.256775
(7, 2)	0.50, 2.50	0.2673	0.0282	0.082351	34.2	0.106328
(7, 3)	0.50, 2.50	0.1570	0.0430	0.090293	47.7	0.140766
(7, 4)	0.50, 2.50	0.1570	0.0441	0.092393	47.8	0.136088
(7, 5)	0.50, 2.50	0.0924	0.0770	0.103880	74.1	0.128467
(7, 6)	0.50, 2.50	0.0368	0.1126	0.117081	96.2	0.121622
(7, 7)	0.50, 2.50	0.0208	0.1212	0.121867	99.5	0.123521
(7, 8)	0.50, 2.50	-.0123	0.1235	0.129311	95.5	0.126972
(7, 9)	0.50, 2.50	-.0216	0.1208	0.131031	92.2	0.129924
(8, 1)	0.75, 2.75	0.2055	0.0319	0.072283	44.1	0.395817
(8, 2)	0.75, 2.75	0.1679	0.0420	0.079319	52.9	0.301338
(8, 3)	0.75, 2.75	0.1091	0.0617	0.090245	68.4	0.178992
(8, 4)	0.75, 2.75	0.0795	0.0785	0.098947	79.3	0.146125
(8, 5)	0.75, 2.75	0.0410	0.1039	0.111626	93.1	0.134649
(8, 6)	0.75, 2.75	0.0172	0.1206	0.120975	99.7	0.128567
(9, 1)	1.00, 3.00	0.1484	0.0538	0.082985	64.9	0.270298
(9, 2)	1.00, 3.00	0.0861	0.0763	0.095767	79.6	0.191874
(9, 3)	1.00, 3.00	0.0473	0.1020	0.109944	92.8	0.142759
(9, 4)	1.00, 3.00	0.0263	0.1141	0.116796	97.7	0.136770
(10, 1)	0.25, 2.50	0.3556	0.0041	0.065738	6.3	0.176746
(10, 2)	0.25, 2.50	0.0193	0.1232	0.123947	99.4	0.124682
(11, 1)	0.75, 3.00	0.2153	0.0292	0.068635	42.5	0.405886
(11, 2)	0.75, 3.00	0.0193	0.1188	0.119639	99.3	0.129064
(11, 3)	0.75, 3.00	0.0151	0.1209	0.121212	99.8	0.129195
(12, 1)	0.00, 0.00	0.0000	0.1250	0.126837	98.5	0.129088

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.0833333 2: 0.0833333 3: 0.1666667 4: 0.1666667
5: 0.1666667 6: 0.1666667 7: 0.0833333 8: 0.0833333

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO:(1, 1, 2, 2).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.105961	82.6	0.088181
(1, 2)	0.75, 1.75	0.0206	0.1232	0.123354	99.9	0.115988
(2, 1)	1.50, 2.50	0.0431	0.1110	0.118240	93.9	0.165500
(2, 2)	1.50, 2.50	0.0210	0.1204	0.122861	98.0	0.145865
(3, 1)	0.25, 2.00	0.2841	0.0145	0.077406	18.7	0.058199
(3, 2)	0.25, 2.00	0.0206	0.1224	0.122618	99.9	0.121651
(3, 3)	0.25, 2.00	0.0164	0.1236	0.123872	99.7	0.122515
(4, 1)	0.75, 2.50	0.1897	0.0387	0.081794	47.3	0.247234
(4, 2)	0.75, 2.50	0.1610	0.0449	0.084817	52.9	0.176187
(4, 3)	0.75, 2.50	0.0869	0.0764	0.099402	76.9	0.141625
(4, 4)	0.75, 2.50	0.0543	0.1015	0.110677	91.7	0.129230
(4, 5)	0.75, 2.50	0.0543	0.1027	0.111249	92.3	0.127497
(4, 6)	0.75, 2.50	0.0408	0.1098	0.114800	95.7	0.128837
(4, 7)	0.75, 2.50	0.0247	0.1189	0.120032	99.1	0.127106
(4, 8)	0.75, 2.50	-.0132	0.1231	0.131128	93.9	0.128883
(5, 1)	1.25, 3.00	0.0965	0.0745	0.100200	74.4	0.217074
(5, 2)	1.25, 3.00	0.0203	0.1185	0.121838	97.3	0.139476
(6, 1)	0.25, 2.25	0.3256	0.0069	0.068847	10.1	0.063691
(6, 2)	0.25, 2.25	0.1648	0.0314	0.084824	37.0	0.081574
(6, 3)	0.25, 2.25	0.0842	0.0808	0.103589	78.0	0.103779
(6, 4)	0.25, 2.25	0.0667	0.0953	0.108945	87.5	0.109405
(6, 5)	0.25, 2.25	0.0371	0.1162	0.118215	98.3	0.117154
(6, 6)	0.25, 2.25	0.0198	0.1232	0.123396	99.8	0.122285
(7, 1)	0.50, 2.50	0.2673	0.0161	0.070163	22.9	0.256349
(7, 2)	0.50, 2.50	0.2673	0.0282	0.077699	36.3	0.100321
(7, 3)	0.50, 2.50	0.1570	0.0430	0.086462	49.8	0.134793
(7, 4)	0.50, 2.50	0.1570	0.0441	0.087473	50.5	0.128840
(7, 5)	0.50, 2.50	0.0924	0.0770	0.100752	76.4	0.124600
(7, 6)	0.50, 2.50	0.0368	0.1126	0.116005	97.1	0.120505
(7, 7)	0.50, 2.50	0.0208	0.1212	0.121523	99.8	0.123172
(7, 8)	0.50, 2.50	-.0123	0.1235	0.130713	94.5	0.128348
(7, 9)	0.50, 2.50	-.0216	0.1208	0.133124	90.8	0.132000
(8, 1)	0.75, 2.75	0.2055	0.0319	0.076722	41.6	0.420127
(8, 2)	0.75, 2.75	0.1679	0.0420	0.082390	51.0	0.313006
(8, 3)	0.75, 2.75	0.1091	0.0617	0.092111	67.0	0.182693
(8, 4)	0.75, 2.75	0.0795	0.0785	0.100005	78.5	0.147688
(8, 5)	0.75, 2.75	0.0410	0.1039	0.111950	92.8	0.135040
(8, 6)	0.75, 2.75	0.0172	0.1206	0.121684	99.1	0.129320
(9, 1)	1.00, 3.00	0.1484	0.0538	0.088023	61.1	0.286708
(9, 2)	1.00, 3.00	0.0861	0.0763	0.099563	76.6	0.199479
(9, 3)	1.00, 3.00	0.0473	0.1020	0.112147	90.9	0.145620
(9, 4)	1.00, 3.00	0.0263	0.1141	0.119085	95.8	0.139450
(10, 1)	0.25, 2.50	0.3556	0.0041	0.063422	6.5	0.170518
(10, 2)	0.25, 2.50	0.0193	0.1232	0.123533	99.8	0.124266
(11, 1)	0.75, 3.00	0.2153	0.0292	0.074702	39.0	0.441765
(11, 2)	0.75, 3.00	0.0193	0.1188	0.120484	98.6	0.129976
(11, 3)	0.75, 3.00	0.0151	0.1209	0.122069	99.1	0.130109
(12, 1)	0.00, 0.00	0.0000	0.1250	0.127824	97.8	0.130093

THE COEFFICIENTS OF THE ESTIMATOR ARE:

1:	0.1000000	2:	0.1000000	3:	0.1000000	4:	0.2000000
5:	0.2000000	6:	0.1000000	7:	0.1000000	8:	0.1000000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO:(1, 1, 1, 2).

DISTRI-BUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.110375	79.3	0.091854
(1, 2)	0.75, 1.75	0.0206	0.1232	0.123418	99.8	0.116048
(2, 1)	1.50, 2.50	0.0431	0.1110	0.121211	91.6	0.169660
(2, 2)	1.50, 2.50	0.0210	0.1204	0.123873	97.2	0.147067
(3, 1)	0.25, 2.00	0.2841	0.0145	0.088997	16.3	0.066914
(3, 2)	0.25, 2.00	0.0206	0.1224	0.122656	99.8	0.121689
(3, 3)	0.25, 2.00	0.0164	0.1236	0.123647	99.9	0.122292
(4, 1)	0.75, 2.50	0.1897	0.0387	0.096437	40.1	0.291495
(4, 2)	0.75, 2.50	0.1610	0.0449	0.098367	45.7	0.204335
(4, 3)	0.75, 2.50	0.0869	0.0764	0.107698	71.0	0.153445
(4, 4)	0.75, 2.50	0.0543	0.1015	0.115172	88.1	0.134479
(4, 5)	0.75, 2.50	0.0543	0.1027	0.115519	88.9	0.132390
(4, 6)	0.75, 2.50	0.0408	0.1098	0.117924	93.1	0.132343
(4, 7)	0.75, 2.50	0.0247	0.1189	0.121451	97.9	0.128608
(4, 8)	0.75, 2.50	-.0132	0.1231	0.129373	95.2	0.127157
(5, 1)	1.25, 3.00	0.0965	0.0745	0.109543	68.0	0.237314
(5, 2)	1.25, 3.00	0.0203	0.1185	0.123300	96.1	0.141150
(6, 1)	0.25, 2.25	0.3256	0.0069	0.085460	8.1	0.079060
(6, 2)	0.25, 2.25	0.1648	0.0314	0.095198	33.0	0.091550
(6, 3)	0.25, 2.25	0.0842	0.0808	0.108308	74.6	0.108507
(6, 4)	0.25, 2.25	0.0667	0.0953	0.112210	85.0	0.112684
(6, 5)	0.25, 2.25	0.0371	0.1162	0.119121	97.5	0.118052
(6, 6)	0.25, 2.25	0.0198	0.1232	0.123259	99.9	0.122149
(7, 1)	0.50, 2.50	0.2673	0.0161	0.088514	18.2	0.323399
(7, 2)	0.50, 2.50	0.2673	0.0282	0.093254	30.2	0.120404
(7, 3)	0.50, 2.50	0.1570	0.0430	0.098059	43.9	0.152872
(7, 4)	0.50, 2.50	0.1570	0.0441	0.098567	44.8	0.145180
(7, 5)	0.50, 2.50	0.0924	0.0770	0.107409	71.7	0.132831
(7, 6)	0.50, 2.50	0.0368	0.1126	0.118186	95.3	0.122771
(7, 7)	0.50, 2.50	0.0208	0.1212	0.122123	99.3	0.123780
(7, 8)	0.50, 2.50	-.0123	0.1235	0.129132	95.7	0.126795
(7, 9)	0.50, 2.50	-.0216	0.1208	0.131078	92.2	0.129972
(8, 1)	0.75, 2.75	0.2055	0.0319	0.093670	34.0	0.512935
(8, 2)	0.75, 2.75	0.1679	0.0420	0.097255	43.2	0.369479
(8, 3)	0.75, 2.75	0.1091	0.0617	0.103224	59.8	0.204734
(8, 4)	0.75, 2.75	0.0795	0.0785	0.108223	72.5	0.159823
(8, 5)	0.75, 2.75	0.0410	0.1039	0.115838	89.7	0.139730
(8, 6)	0.75, 2.75	0.0172	0.1206	0.122667	98.3	0.130365
(9, 1)	1.00, 3.00	0.1484	0.0538	0.101370	53.1	0.330181
(9, 2)	1.00, 3.00	0.0861	0.0763	0.108744	70.1	0.217872
(9, 3)	1.00, 3.00	0.0473	0.1020	0.116827	87.3	0.151697
(9, 4)	1.00, 3.00	0.0263	0.1141	0.121463	93.9	0.142236
(10, 1)	0.25, 2.50	0.3556	0.0041	0.083556	4.9	0.224650
(10, 2)	0.25, 2.50	0.0193	0.1232	0.123287	100.0	0.124018
(11, 1)	0.75, 3.00	0.2153	0.0292	0.092490	31.5	0.546957
(11, 2)	0.75, 3.00	0.0193	0.1188	0.121873	97.5	0.131474
(11, 3)	0.75, 3.00	0.0151	0.1209	0.122951	98.4	0.131048
(12, 1)	0.00, 0.00	0.0000	0.1250	0.126915	98.5	0.129167

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: 0.1250000 2: 0.1250000 3: 0.1250000 4: 0.1250000
5: 0.1250000 6: 0.1250000 7: 0.1250000 8: 0.1250000

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO: (1, 1, 1, 1).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.125000	70.0	0.104025
(1, 2)	0.75, 1.75	0.0206	0.1232	0.125000	98.6	0.117536
(2, 1)	1.50, 2.50	0.0431	0.1110	0.125000	88.8	0.174963
(2, 2)	1.50, 2.50	0.0210	0.1204	0.125000	96.3	0.148405
(3, 1)	0.25, 2.00	0.2841	0.0145	0.125000	11.6	0.093984
(3, 2)	0.25, 2.00	0.0206	0.1224	0.125000	98.0	0.124014
(3, 3)	0.25, 2.00	0.0164	0.1236	0.125000	98.8	0.123631
(4, 1)	0.75, 2.50	0.1897	0.0387	0.125000	30.9	0.377829
(4, 2)	0.75, 2.50	0.1610	0.0449	0.125000	35.9	0.259659
(4, 3)	0.75, 2.50	0.0869	0.0764	0.125000	61.2	0.178097
(4, 4)	0.75, 2.50	0.0543	0.1015	0.125000	81.2	0.145955
(4, 5)	0.75, 2.50	0.0543	0.1027	0.125000	82.2	0.143256
(4, 6)	0.75, 2.50	0.0408	0.1098	0.125000	87.9	0.140284
(4, 7)	0.75, 2.50	0.0247	0.1189	0.125000	95.1	0.132367
(4, 8)	0.75, 2.50	-.0132	0.1231	0.125000	98.5	0.122859
(5, 1)	1.25, 3.00	0.0965	0.0745	0.125000	59.6	0.270800
(5, 2)	1.25, 3.00	0.0203	0.1185	0.125000	94.8	0.143096
(6, 1)	0.25, 2.25	0.3256	0.0069	0.125000	5.6	0.115638
(6, 2)	0.25, 2.25	0.1648	0.0314	0.125000	25.1	0.120210
(6, 3)	0.25, 2.25	0.0842	0.0808	0.125000	64.6	0.125229
(6, 4)	0.25, 2.25	0.0667	0.0953	0.125000	76.3	0.125528
(6, 5)	0.25, 2.25	0.0371	0.1162	0.125000	92.9	0.123878
(6, 6)	0.25, 2.25	0.0198	0.1232	0.125000	98.5	0.123875
(7, 1)	0.50, 2.50	0.2673	0.0161	0.125000	12.9	0.456706
(7, 2)	0.50, 2.50	0.2673	0.0282	0.125000	22.6	0.161393
(7, 3)	0.50, 2.50	0.1570	0.0430	0.125000	34.4	0.194873
(7, 4)	0.50, 2.50	0.1570	0.0441	0.125000	35.3	0.184115
(7, 5)	0.50, 2.50	0.0924	0.0770	0.125000	61.6	0.154587
(7, 6)	0.50, 2.50	0.0368	0.1126	0.125000	90.1	0.129849
(7, 7)	0.50, 2.50	0.0208	0.1212	0.125000	97.0	0.126696
(7, 8)	0.50, 2.50	-.0123	0.1235	0.125000	98.8	0.122738
(7, 9)	0.50, 2.50	-.0216	0.1208	0.125000	96.7	0.123944
(8, 1)	0.75, 2.75	0.2055	0.0319	0.125000	25.5	0.684494
(8, 2)	0.75, 2.75	0.1679	0.0420	0.125000	33.6	0.474885
(8, 3)	0.75, 2.75	0.1091	0.0617	0.125000	49.4	0.247924
(8, 4)	0.75, 2.75	0.0795	0.0785	0.125000	62.8	0.184600
(8, 5)	0.75, 2.75	0.0410	0.1039	0.125000	83.2	0.150781
(8, 6)	0.75, 2.75	0.0172	0.1206	0.125000	96.5	0.132844
(9, 1)	1.00, 3.00	0.1484	0.0538	0.125000	43.1	0.407148
(9, 2)	1.00, 3.00	0.0861	0.0763	0.125000	61.0	0.250443
(9, 3)	1.00, 3.00	0.0473	0.1020	0.125000	81.6	0.162310
(9, 4)	1.00, 3.00	0.0263	0.1141	0.125000	91.2	0.146377
(10, 1)	0.25, 2.50	0.3556	0.0041	0.125000	3.3	0.336079
(10, 2)	0.25, 2.50	0.0193	0.1232	0.125000	98.6	0.125742
(11, 1)	0.75, 3.00	0.2153	0.0292	0.125000	23.3	0.739210
(11, 2)	0.75, 3.00	0.0193	0.1188	0.125000	95.0	0.134847
(11, 3)	0.75, 3.00	0.0151	0.1209	0.125000	96.8	0.133233
(12, 1)	0.00, 0.00	0.0000	0.1250	0.125000	100.0	0.127218

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: -0.0192308 2: 0.0384615 3: 0.1923077 4: 0.2884615
5: 0.2884615 6: 0.1923077 7: 0.0384615 8: -0.0192308

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO: (-1, 2,10,15).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.089318	98.0	0.074331
(1, 2)	0.75, 1.75	0.0206	0.1232	0.137044	89.9	0.128861
(2, 1)	1.50, 2.50	0.0431	0.1110	0.126212	88.0	0.176659
(2, 2)	1.50, 2.50	0.0210	0.1204	0.137392	87.6	0.163118
(3, 1)	0.25, 2.00	0.2841	0.0145	0.026507	54.7	0.019930
(3, 2)	0.25, 2.00	0.0206	0.1224	0.134826	90.8	0.133763
(3, 3)	0.25, 2.00	0.0164	0.1236	0.138488	89.2	0.136971
(4, 1)	0.75, 2.50	0.1897	0.0387	0.039096	98.9	0.118172
(4, 2)	0.75, 2.50	0.1610	0.0449	0.045220	99.3	0.093934
(4, 3)	0.75, 2.50	0.0869	0.0764	0.076795	99.5	0.109415
(4, 4)	0.75, 2.50	0.0543	0.1015	0.104124	97.4	0.121579
(4, 5)	0.75, 2.50	0.0543	0.1027	0.105263	97.6	0.120636
(4, 6)	0.75, 2.50	0.0408	0.1098	0.114694	95.7	0.128718
(4, 7)	0.75, 2.50	0.0247	0.1189	0.128524	92.5	0.136099
(4, 8)	0.75, 2.50	-.0132	0.1231	0.160390	76.8	0.157643
(5, 1)	1.25, 3.00	0.0965	0.0745	0.088012	84.7	0.190669
(5, 2)	1.25, 3.00	0.0203	0.1185	0.135161	87.7	0.154728
(6, 1)	0.25, 2.25	0.3256	0.0069	0.013517	51.4	0.012505
(6, 2)	0.25, 2.25	0.1648	0.0314	0.040209	78.1	0.038668
(6, 3)	0.25, 2.25	0.0842	0.0808	0.083126	97.2	0.083278
(6, 4)	0.25, 2.25	0.0667	0.0953	0.096841	98.4	0.097250
(6, 5)	0.25, 2.25	0.0371	0.1162	0.122002	95.2	0.120907
(6, 6)	0.25, 2.25	0.0198	0.1232	0.137008	89.9	0.135774
(7, 1)	0.50, 2.50	0.2673	0.0161	0.017379	92.5	0.063498
(7, 2)	0.50, 2.50	0.2673	0.0282	0.029935	94.2	0.038651
(7, 3)	0.50, 2.50	0.1570	0.0430	0.045646	94.3	0.071161
(7, 4)	0.50, 2.50	0.1570	0.0441	0.047267	93.4	0.069620
(7, 5)	0.50, 2.50	0.0924	0.0770	0.077550	99.3	0.095906
(7, 6)	0.50, 2.50	0.0368	0.1126	0.116852	96.4	0.121385
(7, 7)	0.50, 2.50	0.0208	0.1212	0.132112	91.8	0.133905
(7, 8)	0.50, 2.50	-.0123	0.1235	0.159203	77.6	0.156323
(7, 9)	0.50, 2.50	-.0216	0.1208	0.166729	72.5	0.165321
(8, 1)	0.75, 2.75	0.2055	0.0319	0.032441	98.3	0.177644
(8, 2)	0.75, 2.75	0.1679	0.0420	0.042571	98.6	0.161729
(8, 3)	0.75, 2.75	0.1091	0.0617	0.062229	99.2	0.123425
(8, 4)	0.75, 2.75	0.0795	0.0785	0.079536	98.7	0.117459
(8, 5)	0.75, 2.75	0.0410	0.1039	0.107735	96.5	0.129955
(8, 6)	0.75, 2.75	0.0172	0.1206	0.133485	90.4	0.141861
(9, 1)	1.00, 3.00	0.1484	0.0538	0.057134	94.2	0.186095
(9, 2)	1.00, 3.00	0.0861	0.0763	0.081589	93.5	0.163468
(9, 3)	1.00, 3.00	0.0473	0.1020	0.110354	92.4	0.143293
(9, 4)	1.00, 3.00	0.0263	0.1141	0.128568	88.7	0.150556
(10, 1)	0.25, 2.50	0.3556	0.0041	0.007221	57.1	0.019415
(10, 2)	0.25, 2.50	0.0193	0.1232	0.137420	89.7	0.138236
(11, 1)	0.75, 3.00	0.2153	0.0292	0.030112	96.9	0.178070
(11, 2)	0.75, 3.00	0.0193	0.1188	0.130630	90.9	0.140921
(11, 3)	0.75, 3.00	0.0151	0.1209	0.134742	89.8	0.143616
(12, 1)	0.00, 0.00	0.0000	0.1250	0.150532	83.0	0.153203

THE COEFFICIENTS OF THE ESTIMATOR ARE:
1: -0.0192308 2: 0.0384615 3: 0.2884615 4: 0.1923077
5: 0.1923077 6: 0.2884615 7: 0.0384615 8: -0.0192308

THE FIRST FOUR COEFFICIENTS ARE IN THE RATIO:(-1, 2,15,10).

DISTRIBUTION	CUTPOINTS	PROB DIFF	VAR OF BEST EST	VAR OF EST	EFFIC OF EST	VAR OF EST FOR 95 PERC SCALING
(1, 1)	0.75, 1.75	0.1003	0.0876	0.092423	94.7	0.076914
(1, 2)	0.75, 1.75	0.0206	0.1232	0.135449	91.0	0.127361
(2, 1)	1.50, 2.50	0.0431	0.1110	0.122339	90.7	0.171238
(2, 2)	1.50, 2.50	0.0210	0.1204	0.133993	89.9	0.159082
(3, 1)	0.25, 2.00	0.2841	0.0145	0.034383	42.2	0.025852
(3, 2)	0.25, 2.00	0.0206	0.1224	0.133885	91.5	0.132830
(3, 3)	0.25, 2.00	0.0164	0.1236	0.137071	90.1	0.135569
(4, 1)	0.75, 2.50	0.1897	0.0387	0.039389	98.2	0.119057
(4, 2)	0.75, 2.50	0.1610	0.0449	0.045436	98.8	0.094383
(4, 3)	0.75, 2.50	0.0869	0.0764	0.076734	99.6	0.109328
(4, 4)	0.75, 2.50	0.0543	0.1015	0.103241	98.3	0.120548
(4, 5)	0.75, 2.50	0.0543	0.1027	0.104415	98.4	0.119664
(4, 6)	0.75, 2.50	0.0408	0.1098	0.113431	96.8	0.127301
(4, 7)	0.75, 2.50	0.0247	0.1189	0.126675	93.9	0.134140
(4, 8)	0.75, 2.50	-.0132	0.1231	0.156088	78.9	0.153415
(5, 1)	1.25, 3.00	0.0965	0.0745	0.084938	87.7	0.184011
(5, 2)	1.25, 3.00	0.0203	0.1185	0.131608	90.1	0.150660
(6, 1)	0.25, 2.25	0.3256	0.0069	0.017868	38.9	0.016529
(6, 2)	0.25, 2.25	0.1648	0.0314	0.045818	68.5	0.044062
(6, 3)	0.25, 2.25	0.0842	0.0808	0.087152	92.7	0.087312
(6, 4)	0.25, 2.25	0.0667	0.0953	0.099897	95.4	0.100319
(6, 5)	0.25, 2.25	0.0371	0.1162	0.122806	94.6	0.121704
(6, 6)	0.25, 2.25	0.0198	0.1232	0.135800	90.7	0.134578
(7, 1)	0.50, 2.50	0.2673	0.0161	0.018494	86.9	0.067571
(7, 2)	0.50, 2.50	0.2673	0.0282	0.031505	89.5	0.040678
(7, 3)	0.50, 2.50	0.1570	0.0430	0.048253	89.2	0.075226
(7, 4)	0.50, 2.50	0.1570	0.0441	0.050152	88.0	0.073870
(7, 5)	0.50, 2.50	0.0924	0.0770	0.079788	96.5	0.098674
(7, 6)	0.50, 2.50	0.0368	0.1126	0.116674	96.5	0.121199
(7, 7)	0.50, 2.50	0.0208	0.1212	0.130836	92.7	0.132611
(7, 8)	0.50, 2.50	-.0123	0.1235	0.154875	79.8	0.152073
(7, 9)	0.50, 2.50	-.0216	0.1208	0.161290	74.9	0.159928
(8, 1)	0.75, 2.75	0.2055	0.0319	0.031921	99.9	0.174798
(8, 2)	0.75, 2.75	0.1679	0.0420	0.041994	100.0	0.159538
(8, 3)	0.75, 2.75	0.1091	0.0617	0.061904	99.7	0.122779
(8, 4)	0.75, 2.75	0.0795	0.0785	0.079150	99.2	0.116888
(8, 5)	0.75, 2.75	0.0410	0.1039	0.107104	97.0	0.129194
(8, 6)	0.75, 2.75	0.0172	0.1206	0.131185	91.9	0.139417
(9, 1)	1.00, 3.00	0.1484	0.0538	0.055432	97.1	0.180552
(9, 2)	1.00, 3.00	0.0861	0.0763	0.079520	95.9	0.159322
(9, 3)	1.00, 3.00	0.0473	0.1020	0.107808	94.6	0.139986
(9, 4)	1.00, 3.00	0.0263	0.1141	0.125284	91.0	0.146710
(10, 1)	0.25, 2.50	0.3556	0.0041	0.009134	45.1	0.024558
(10, 2)	0.25, 2.50	0.0193	0.1232	0.136333	90.4	0.137143
(11, 1)	0.75, 3.00	0.2153	0.0292	0.029431	99.1	0.174048
(11, 2)	0.75, 3.00	0.0193	0.1188	0.128429	92.5	0.138547
(11, 3)	0.75, 3.00	0.0151	0.1209	0.132335	91.4	0.141051
(12, 1)	0.00, 0.00	0.0000	0.1250	0.147219	84.9	0.149832

THE FACTORS BY WHICH FORTY-EIGHT DISTRIBUTIONS MUST BE SCALED TO MAKE THEIR 95TH PERCENTILES 1.645, AND THE VARIANCES OF THE SCALED DISTRIBUTIONS

DISTRIBUTION	95TH PERCENTILE	SCALE FACTOR	VARIANCE OF SCALED DISTRIBUTION
(1, 1)	1.8032	0.9123	0.8322
(1, 2)	1.6964	0.9697	0.9403
(2, 1)	1.3904	1.1831	1.3997
(2, 2)	1.5097	1.0896	1.1872
(3, 1)	1.8971	0.8671	0.7519
(3, 2)	1.6515	0.9961	0.9921
(3, 3)	1.6541	0.9945	0.9890
(4, 1)	0.9462	1.7386	3.0226
(4, 2)	1.1414	1.4413	2.0773
(4, 3)	1.3781	1.1936	1.4248
(4, 4)	1.5223	1.0806	1.1676
(4, 5)	1.5366	1.0705	1.1460
(4, 6)	1.5528	1.0594	1.1223
(4, 7)	1.5986	1.0290	1.0589
(4, 8)	1.6593	0.9914	0.9829
(5, 1)	1.1176	1.4719	2.1664
(5, 2)	1.5375	1.0699	1.1448
(6, 1)	1.7103	0.9618	0.9251
(6, 2)	1.6775	0.9807	0.9617
(6, 3)	1.6435	1.0009	1.0018
(6, 4)	1.6415	1.0021	1.0042
(6, 5)	1.6524	0.9955	0.9910
(6, 6)	1.6525	0.9955	0.9910
(7, 1)	0.8606	1.9115	3.6536
(7, 2)	1.4477	1.1363	1.2911
(7, 3)	1.3175	1.2486	1.5590
(7, 4)	1.3554	1.2136	1.4729
(7, 5)	1.4792	1.1121	1.2367
(7, 6)	1.6140	1.0192	1.0388
(7, 7)	1.6340	1.0068	1.0136
(7, 8)	1.6601	0.9909	0.9819
(7, 9)	1.6520	0.9958	0.9916
(8, 1)	0.7030	2.3401	5.4760
(8, 2)	0.8440	1.9491	3.7991
(8, 3)	1.1681	1.4083	1.9834
(8, 4)	1.3536	1.2152	1.4768
(8, 5)	1.4978	1.0983	1.2062
(8, 6)	1.5957	1.0309	1.0627
(9, 1)	0.9115	1.8048	3.2572
(9, 2)	1.1622	1.4155	2.0035
(9, 3)	1.4436	1.1395	1.2985
(9, 4)	1.5201	1.0821	1.1710
(10, 1)	1.0032	1.6397	2.6886
(10, 2)	1.6401	1.0030	1.0059
(11, 1)	0.6765	2.4318	5.9137
(11, 2)	1.5838	1.0386	1.0788
(11, 3)	1.5934	1.0324	1.0659
(12, 1)	1.6306	1.0088	1.0177

APPENDIX 6

THE STEP DISTRIBUTIONS

This appendix gives the parameters of seven three-step distributions and four five-step distributions.

All the distributions are symmetric and have mean 0 and variance 1. Parameters not given can be found from considerations of symmetry.

DISTRIBUTION 1

TAIL PROBABILITY: 0.25 HEIGHT RATIO: 2.00

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-1.9867985356	0.157288217402
2	-1.19207912135	0.314576434805

DISTRIBUTION 2

TAIL PROBABILITY: 0.25 HEIGHT RATIO: 10.00

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-2.88296295673	0.056365621909
2	-0.66529914386	0.56365621909

DISTRIBUTION 3

TAIL PROBABILITY: 0.25 HEIGHT RATIO: 100.00

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-3.40911364873	0.0377664147536
2	-0.099294572292	3.77664147537

DISTRIBUTION 4

TAIL PROBABILITY: 0.25 HEIGHT RATIO: 1000.00

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-3.45887089448	0.0362473777784
2	-0.0103455759556	36.2473777789

DISTRIBUTION 5

TAIL PROBABILITY: 0.25 HEIGHT RATIO: 100000.00

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-3.46404965009	0.0360860156832
2	-1.03918371952E-4	3608.60156831

DISTRIBUTION 6

TAIL PROBABILITY: 0.50 HEIGHT RATIO: 0.01

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-1.23236424015	20.4890722866
2	-1.22016261402	0.204890722867

DISTRIBUTION 7

TAIL PROBABILITY: 0.05 HEIGHT RATIO: 1000.0

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-7.648685986	0.00333063745148
2	-0.142615342234	3.33063745149

DISTRIBUTION 8

MIDDLE PROBABILITY: 0.05 MIDDLE HEIGHT RATIO: 100.0
TAIL PROBABILITY: 0.05 TAIL HEIGHT RATIO: 1000.0

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-6.0501650442	0.00461970868488
2	-0.6385684036	4.6197086849
3	-0.541159664066	0.0461970868489

DISTRIBUTION 9

MIDDLE PROBABILITY: 0.10 MIDDLE HEIGHT RATIO: 100.0
TAIL PROBABILITY: 0.05 TAIL HEIGHT RATIO: 1000.0

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-4.66094 26378	0.0065 276495 259
2	-0.83108015808	6.527649526
3	-0.76597249593	0.065 27649526

DISTRIBUTION 10

MIDDLE PROBABILITY: 0.25 MIDDLE HEIGHT RATIO: 100.0
TAIL PROBABILITY: 0.05 TAIL HEIGHT RATIO: 1000.0

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-2.99022953781	0.012657891149
2	-1.01517700293	12.6578911489
3	-0.98752626744	0.12657891149

DISTRIBUTION 11

MIDDLE PROBABILITY: 0.45 MIDDLE HEIGHT RATIO: 100.0
TAIL PROBABILITY: 0.05 TAIL HEIGHT RATIO: 1000.0

R	LEFT ENDPOINT OF RTH REGION	PROBABILITY DENSITY IN RTH REGION
1	-2.39765849536	0.0199152632006
2	-1.14233991141	19.9152632004
3	-1.12978672556	0.19915263200 3

APPENDIX 7

THE COEFFICIENTS AND VARIANCES OF THE BEST LINEAR LOCATION ESTIMATORS FOR THE STEP DISTRIBUTIONS FOR SAMPLE SIZES OF 4, 8, 16 AND 32.

DISTRIBUTION 1

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	0.419385955	0.359329573	0.352441844	0.411460014
2	0.080614045	0.064127876	-0.010031684	-0.025894674
3		0.055107474	0.039754858	-0.025022909
4		0.021435077	0.060002341	-0.010119592
5			0.038372095	0.012342677
6			0.014593368	0.041121878
7			0.003931816	0.041835008
8			0.000935361	0.028860666
9				0.016782981
10				0.006390808
11				0.001147508
12				0.000299168
13				0.001305577
14				-0.000168773
15				-0.001014663
16				0.000674327

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.228596914	0.100319553	0.042202240	0.014990537

DISTRIBUTION 2

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	0.029341988	-0.001816716	0.016308984	0.037454682
2	0.470658012	-0.037582158	-0.038619315	-0.014252464
3		0.165534803	-0.040142924	-0.020198033
4		0.373864071	0.009974982	-0.025056686
5			0.169870523	-0.027584553
6			0.238133658	-0.021352619
7			0.112174055	0.013779475
8			0.032300036	0.103595440
9				0.188992956
10				0.157317949
11				0.073116801
12				0.024746980
13				0.008369872
14				0.001279908
15				-0.001569801
16				0.001360093

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.198218279	0.075849069	0.031309543	0.012022972

DISTRIBUTION 3

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	-0.016688617	-0.004205837	-0.001709360	-0.000089681
2	0.516688617	-0.012915240	-0.003601774	-0.001193430
3		-0.025361716	-0.004980037	-0.001786500
4		0.542482793	-0.006403419	-0.002302716
5			-0.007061965	-0.002752954
6			0.003250123	-0.003173216
7			0.107126560	-0.003552079
8			0.413379872	-0.003664912
9				-0.001745833
10				0.013310290
11				0.083314701
12				0.193813590
13				0.156023396
14				0.057197679
15				0.012915668
16				0.003685997

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.107156248	0.008447289	0.000808228	0.000297106

DISTRIBUTION 4

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	-0.010704562	-0.000101805	-0.000185934	-0.000050049
2	0.510704562	-0.004809014	-0.000363209	-0.000114830
3		-0.018096601	-0.000498788	-0.000173691
4		0.523007420	-0.000681484	-0.000225126
5			-0.001182929	-0.000269712
6			-0.004368662	-0.000311022
7			-0.015226032	-0.000352716
8			0.522507038	-0.000397756
9				-0.000446120
10				-0.000473782
11				-0.000274033
12				0.002073010
13				0.023289989
14				0.131720205
15				0.223731982
16				0.122273651

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.101465980	0.006612878	0.000074651	0.000003920

DISTRIBUTION 5

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	-0.009923031	0.000383753	0.000017145	-0.000000532
2	0.509923031	-0.003898633	-0.000018487	-0.000001151
3		-0.016794961	-0.000017312	-0.000001738
4		0.520309841	-0.000051174	-0.000002246
5			-0.000379821	-0.000002686
6			-0.003433782	-0.000003098
7			-0.015769500	-0.000003520
8			0.519652931	-0.000003984
9				-0.000004553
10				-0.000005583
11				-0.000009908
12				-0.000044636
13				-0.000389316
14				-0.003725464
15				-0.014770176
16				0.518968591

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.100959964	0.006531539	0.000065831	0.000000021

DISTRIBUTION 6

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	0.563528873	0.551563438	0.535534003	0.520114637
2	-0.063528873	-0.037710913	-0.028944026	-0.015231729
3		-0.008886027	-0.004429430	-0.002590597
4		-0.004966498	-0.001121538	-0.000866093
5			-0.000424982	-0.000463823
6			-0.000239033	-0.000282723
7			-0.000191476	-0.000206257
8			-0.000183518	-0.000146642
9				-0.000111658
10				-0.000082548
11				-0.000057128
12				-0.000036712
13				-0.000021210
14				-0.000010595
15				-0.000004647
16				-0.000002272

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.129197960	0.014243658	0.000440871	0.000002214

DISTRIBUTION 7

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	-0.010507237	-0.003439665	-0.001696367	-0.000804643
2	0.510507237	-0.003686785	-0.002329729	-0.001085054
3		0.079827944	-0.001184751	-0.001272290
4		0.427298505	0.049729036	-0.001097168
5			0.326412098	0.005519321
6			0.121208066	0.099797355
7			0.007560309	0.290872508
8			0.000301338	0.096946825
9				0.010299684
10				0.000773342
11				0.000047526
12				0.000002480
13				0.000000110
14				0.000000005
15				-0.000000001
16				0.000000000

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.022349202	0.001923829	0.000721680	0.000258764

DISTRIBUTION 8

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	0.094320655	-0.009461878	-0.002705895	-0.000532855
2	0.405679345	0.326651978	-0.004847984	-0.000749230
3		0.202777386	0.156088876	-0.000996744
4		-0.019967486	0.373647523	-0.001730140
5			-0.003422487	0.009717743
6			-0.008386032	0.309635795
7			-0.005470514	0.212914571
8			-0.004903487	-0.006123009
9				-0.004833676
10				-0.003214059
11				-0.002846284
12				-0.002582100
13				-0.002358305
14				-0.002190896
15				-0.002082045
16				-0.002028769

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.199661080	0.054976352	0.005842277	0.000160064

DISTRIBUTION 9

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	0.268102374	0.014287795	-0.003096983	-0.000531561
2	0.231897626	0.444975402	-0.000433398	-0.000768466
3		0.061090412	0.317000354	-0.001160173
4		-0.020353608	0.219086690	-0.002493373
5			-0.019495826	0.043608901
6			-0.006906548	0.421974789
7			-0.003344705	0.068053588
8			-0.002809584	-0.014636765
9				-0.004832687
10				-0.002146363
11				-0.001526103
12				-0.001302694
13				-0.001168988
14				-0.001074792
15				-0.001012981
16				-0.000982331

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.249452764	0.060847417	0.006289433	0.000132875

DISTRIBUTION 10

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	0.468451863	0.130880386	-0.003693685	-0.000555886
2	0.031548137	0.420145272	0.059095056	-0.001029114
3		-0.025054140	0.446271141	-0.002565698
4		-0.025971518	0.029603027	0.011926922
5			-0.018461901	0.348063750
6			-0.007002740	0.182477345
7			-0.003406445	-0.021706621
8			-0.002404453	-0.009039964
9				-0.003062425
10				-0.001422225
11				-0.000889051
12				-0.000654480
13				-0.000513038
14				-0.000408704
15				-0.000331722
16				-0.000289090

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.183051655	0.050776305	0.005437812	0.000184894

DISTRIBUTION 11

THE COEFFICIENTS,W[I],FOR SAMPLE SIZE N

I	W[I] FOR N=4	W[I] FOR N=8	W[I] FOR N=16	W[I] FOR N=32
1	0.526504995	0.293639317	0.002413100	-0.000968574
2	-0.026504995	0.276599810	0.262811496	-0.003053466
3		-0.037721694	0.291567849	0.012400621
4		-0.032517433	-0.019978960	0.304668244
5			-0.016208497	0.230809178
6			-0.008533661	-0.019591350
7			-0.006314046	-0.013203481
8			-0.005757281	-0.004618126
9				-0.001800312
10				-0.000942389
11				-0.000716889
12				-0.000684519
13				-0.000671579
14				-0.000617698
15				-0.000535506
16				-0.000474155

THE VARIANCES OF THE ESTIMATORS

N=4	N=8	N=16	N=32
0.160670018	0.046823144	0.008230472	0.000656004

APPENDIX 8

THE EXPECTED VALUES OF THE ORDER STATISTICS FOR SAMPLES OF SIZE 4, 8, 16 AND 32 FROM THE STEP DISTRIBUTIONS

DISTRIBUTION 1

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-1.041323022	-1.386576206	-1.632130913	-1.795314225
2	-0.328093391	-0.922890048	-1.324378393	-1.609369351
3		-0.537220013	-1.070243133	-1.436083838
4		-0.177481077	-0.854405917	-1.281491830
5			-0.657717723	-1.146921402
6			-0.468142013	-1.028938569
7			-0.280598324	-0.922029502
8			-0.093509499	-0.821213247
9				-0.723206748
10				-0.626336183
11				-0.529875227
12				-0.433506029
13				-0.337139360
14				-0.240809898
15				-0.144512276
16				-0.048181677

DISTRIBUTION 2

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.972545204	-1.445112562	-1.936388725	-2.351249212
2	-0.228674924	-0.693069487	-1.225464290	-1.847358123
3		-0.332893467	-0.783853906	-1.407060445
4		-0.102966726	-0.534610703	-1.060642034
5			-0.381551088	-0.814804547
6			-0.264216762	-0.652304583
7			-0.157089068	-0.545371546
8			-0.052246113	-0.469051963
9				-0.406935138
10				-0.350466022
11				-0.295944506
12				-0.241987667
13				-0.188166719
14				-0.134397570
15				-0.080652524
16				-0.026890216

DISTRIBUTION 3

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.802298205	-1.342760401	-2.010852332	-2.616422315
2	-0.110636521	-0.402316297	-0.999463905	-1.869407963
3		-0.105211347	-0.430215925	-1.226797954
4		-0.021940297	-0.176782547	-0.738309132
5			-0.081256468	-0.414949556
6			-0.044381602	-0.228411211
7			-0.024263248	-0.133088302
8			-0.007895816	-0.088026204
9				-0.066297896
10				-0.053830774
11				-0.044540594
12				-0.036196822
13				-0.028099127
14				-0.020061271
15				-0.012037645
16				-0.004013362

DISTRIBUTION 4

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.768279242	-1.314216557	-2.003520819	-2.633051625
2	-0.091240773	-0.353319883	-0.954920926	-1.855256167
3		-0.068816735	-0.371178382	-1.187229403
4		-0.009134141	-0.119476804	-0.681242189
5			-0.033795703	-0.348868718
6			-0.009779278	-0.160346618
7			-0.003380304	-0.067725566
8			-0.000924974	-0.027947869
9				-0.012704423
10				-0.007196972
11				-0.005027751
12				-0.003855398
13				-0.002943943
14				-0.002093013
15				-0.001254645
16				-0.000418208

DISTRIBUTION 5

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.764240865	-1.310721770	-2.002358556	-2.634547791
2	-0.088993397	-0.347623181	-0.949641353	-1.853332287
3		-0.064615972	-0.364322885	-1.182485598
4		-0.007658415	-0.112860656	-0.674564301
5			-0.028326541	-0.341205924
6			-0.005794225	-0.152485395
7			-0.000975675	-0.060190099
8			-0.000122327	-0.021027073
9				-0.006532600
10				-0.001827237
11				-0.000478142
12				-0.000131566
13				-0.000047552
14				-0.000024133
15				-0.000013082
16				-0.000004259

DISTRIBUTION 6

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.994222815	-1.186632512	-1.227356760	-1.230874230
2	-0.388030775	-1.020023582	-1.210242369	-1.229262866
3		-0.692834878	-1.160842923	-1.227004345
4		-0.244879567	-1.057635315	-1.222516717
5			-0.891654353	-1.212456318
6			-0.671291933	-1.191621891
7			-0.414807883	-1.154027870
8			-0.140028464	-1.094885803
9				-1.012400008
10				-0.908299650
11				-0.786907675
12				-0.653464020
13				-0.512655745
14				-0.367883545
15				-0.221230705
16				-0.073810026

DISTRIBUTION 7

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.455743864	-0.824475909	-1.460097058	-2.492054443
2	-0.039049508	-0.123982115	-0.272452078	-0.670653280
3		-0.051561780	-0.110641754	-0.217551760
4		-0.016717856	-0.080348694	-0.126991171
5			-0.061858465	-0.106149992
6			-0.044155091	-0.095677340
7			-0.026492069	-0.086445367
8			-0.008830673	-0.077336099
9				-0.068237056
10				-0.059138741
11				-0.050040471
12				-0.040942204
13				-0.031843936
14				-0.022745669
15				-0.013647401
16				-0.004549134

DISTRIBUTION 8

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.797018940	-1.125880849	-1.588405501	-2.332426262
2	-0.228724171	-0.586976079	-0.731876305	-1.019264027
3		-0.417179646	-0.613149605	-0.692594585
4		-0.158059347	-0.581832247	-0.627302191
5			-0.538151182	-0.612265365
6			-0.453381398	-0.604654882
7			-0.309721607	-0.597737178
8			-0.110879386	-0.590225758
9				-0.580793805
10				-0.566756697
11				-0.543354243
12				-0.503703301
13				-0.440076551
14				-0.346721750
15				-0.223332699
16				-0.077269110

DISTRIBUTION 9

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.894162621	-1.172503941	-1.503272617	-2.029847006
2	-0.302401633	-0.765481056	-0.896805985	-1.100503737
3		-0.560470058	-0.810854970	-0.869314858
4		-0.213141271	-0.780604923	-0.823103476
5			-0.725713742	-0.812442765
6			-0.612327845	-0.806965384
7			-0.418067911	-0.801714286
8			-0.149534719	-0.795251653
9				-0.785443966
10				-0.768150696
11				-0.736534095
12				-0.681375669
13				-0.593059563
14				-0.465174526
15				-0.298469206
16				-0.103033330

DISTRIBUTION 10

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.959992038	-1.182074664	-1.361686635	-1.633378663
2	-0.361904745	-0.918860571	-1.047439688	-1.154117282
3		-0.669762471	-0.995496077	-1.034884696
4		-0.25066442O	-0.955345940	-1.010995246
5			-0.870705688	-1.005220025
6			-0.713366880	-1.001369675
7			-0.473031259	-0.995596371
8			-0.166163393	-0.984649776
9				-0.963556096
10				-0.925551406
11				-0.863214964
12				-0.770424186
13				-0.644350491
14				-0.486601797
15				-0.303093175
16				-0.102941602

DISTRIBUTION 11

THE EXPECTED VALUES,E[X[I]],OF THE ORDER STATISTICS
FOR SAMPLE SIZE N

	E[X[I]] FOR N=4	E[X[I]] FOR N=8	E[X[I]] FOR N=16	E[X[I]] FOR N=32
1	-0.997126433	-1.225812835	-1.361440914	-1.535257443
2	-0.372780358	-0.973869658	-1.153618341	-1.230595388
3		-0.669887144	-1.093187006	-1.154486720
4		-0.239712171	-1.008912309	-1.137905613
5			-0.865765605	-1.129843875
6			-0.662801878	-1.116606994
7			-0.414672695	-1.091347563
8			-0.140886438	-1.048215439
9				-0.983027013
10				-0.894597518
11				-0.785010594
12				-0.658643970
13				-0.520610327
14				-0.375434843
15				-0.226415440
16				-0.075636158

APPENDIX 9

THE COVARIANCES OF THE ORDER STATISTICS FOR SAMPLES OF SIZE 4, 8, 16 AND 32 FROM THE STEP DISTRIBUTIONS

Note that the covariances not given can be found from considerations of symmetry.

COVARIANCE[X[R],X[S]] FOR SAMPLE SIZE 4

[R, S]	DISTRIBUTION 1	DISTRIBUTION 2	DISTRIBUTION 3	DISTRIBUTION 4
[1, 1]	0.375597023	0.746701934	1.171152354	1.231094043
[1, 2]	0.244656365	0.235479837	0.190451303	0.181507865
[1, 3]	0.162097336	0.131582792	0.058497497	0.045318559
[1, 4]	0.091292369	0.120905008	0.115518825	0.111820312
[2, 2]	0.432404069	0.255161673	0.172924797	0.170328085
[2, 3]	0.287199138	0.143106130	0.042506424	0.033104712

[R, S]	DISTRIBUTION 5	DISTRIBUTION 6	DISTRIBUTION 7	DISTRIBUTION 8
[1, 1]	1.237805836	0.226637555	1.750289117	1.019676776
[1, 2]	0.180456890	0.214625487	0.066319872	0.149268443
[1, 3]	0.043788085	0.131761079	0.014365893	0.091103322
[1, 4]	0.111349844	0.041677904	0.042900259	0.063301487
[2, 2]	0.170210240	0.634315556	0.040483549	0.292769287
[2, 3]	0.032144130	0.404595853	0.004955545	0.143508921

[R, S]	DISTRIBUTION 9	DISTRIBUTION 10	DISTRIBUTION 11
[1, 1]	0.621450684	0.323127137	0.271851399
[1, 2]	0.180811299	0.203753440	0.217471613
[1, 3]	0.110444940	0.120883404	0.136478476
[1, 4]	0.057063017	0.045768754	0.051130273
[2, 2]	0.487575776	0.624313107	0.594922282
[2, 3]	0.251398045	0.357517314	0.374195869

COVARIANCE[X[R],X[S]] FOR SAMPLE SIZE 8

[R, S]	DISTRIBUTION 1	DISTRIBUTION 2	DISTRIBUTION 3	DISTRIBUTION 4
[1, 1]	0.191958579	0.739599435	1.415230723	1.514331859
[1, 2]	0.135502758	0.301336827	0.451188503	0.469370682
[1, 3]	0.105211648	0.136878993	0.106802589	0.097593597
[1, 4]	0.085394902	0.088036717	0.028504075	0.015415241
[1, 5]	0.068240378	0.068523435	0.017666237	0.006653735
[1, 6]	0.052255776	0.059607962	0.029597915	0.022484576
[1, 7]	0.037853613	0.064362010	0.070331369	0.069352065
[1, 8]	0.022974358	0.066704122	0.107511910	0.112585738
[2, 2]	0.223364756	0.347045403	0.503230658	0.526389504
[2, 3]	0.173241462	0.146814055	0.119892090	0.116337413
[2, 4]	0.140507390	0.088009231	0.026680601	0.017923565
[2, 5]	0.112274800	0.067099484	0.012428791	0.004483461
[2, 6]	0.086009712	0.057991304	0.018396491	0.012239888
[2, 7]	0.062351844	0.062152767	0.044046902	0.040151965
[3, 3]	0.238118560	0.134956774	0.091932775	0.091467338
[3, 4]	0.193401066	0.080126498	0.018424652	0.014789283
[3, 5]	0.154594729	0.060919156	0.006163255	0.002351012
[3, 6]	0.118499245	0.053348973	0.007883786	0.003637214
[4, 4]	0.252133616	0.088282550	0.013191142	0.010992026
[4, 5]	0.201622570	0.068498629	0.004818963	0.002455319

[R, S]	DISTRIBUTION 5	DISTRIBUTION 6	DISTRIBUTION 7	DISTRIBUTION 8
[1, 1]	1.525499122	0.028741284	3.133194838	1.637751674
[1, 2]	0.471349428	0.037230553	0.221873527	0.136421392
[1, 3]	0.096467302	0.038786450	0.025202890	0.063620511
[1, 4]	0.013859882	0.034328298	0.014452207	0.085098715
[1, 5]	0.005347189	0.025963424	0.011407965	0.080227622
[1, 6]	0.021632944	0.015845332	0.008943985	0.048063069
[1, 7]	0.069207465	0.006802471	0.012623328	0.021132043
[1, 8]	0.113140650	0.001516490	0.061162828	0.035757816
[2, 2]	0.529003781	0.148455834	0.159487908	0.126033494
[2, 3]	0.115944131	0.160800421	0.010446301	0.051914288
[2, 4]	0.016941179	0.144856515	0.003132926	0.044403866
[2, 5]	0.003589498	0.111058641	0.002358479	0.032130245
[2, 6]	0.011539686	0.068832666	0.001850561	0.016976031
[2, 7]	0.039690183	0.030055359	0.002626863	0.008745725
[3, 3]	0.091522813	0.343871837	0.006913005	0.150905278
[3, 4]	0.014479608	0.316851154	0.001938789	0.123681124
[3, 5]	0.001999329	0.246329013	0.001438346	0.075928146
[3, 6]	0.003203409	0.154948245	0.001149011	0.034103640
[4, 4]	0.010906974	0.490400047	0.002334056	0.274139334
[4, 5]	0.002317785	0.386641018	0.001792884	0.173368935

[R, S]	DISTRIBUTION 9	DISTRIBUTION 10	DISTRIBUTION 11
[1, 1]	0.828537662	0.233990936	0.116890812
[1, 2]	0.083834040	0.045315389	0.049304764
[1, 3]	0.061883422	0.051581014	0.056677626
[1, 4]	0.084370217	0.062466408	0.057667656
[1, 5]	0.078590285	0.054240698	0.046869270
[1, 6]	0.046412835	0.032012732	0.029475523
[1, 7]	0.018247438	0.012421675	0.013238645
[1, 8]	0.020063660	0.007785406	0.006296928
[2, 2]	0.114490908	0.110304737	0.132314770
[2, 3]	0.082969668	0.117396402	0.142789416
[2, 4]	0.070776243	0.105502900	0.130690823
[2, 5]	0.047521093	0.071565193	0.098848325
[2, 6]	0.023119990	0.034885325	0.058609650
[2, 7]	0.009820715	0.012590067	0.025021121
[3, 3]	0.258382586	0.325806975	0.317568389
[3, 4]	0.223073070	0.307381403	0.297029220
[3, 5]	0.137320170	0.210365576	0.227425682
[3, 6]	0.059639770	0.101198266	0.136489274
[4, 4]	0.478306217	0.576877673	0.475976103
[4, 5]	0.314521540	0.421176518	0.372189418

COVARIANCE[X[R],X[S]] FOR SAMPLE SIZE 16

[R, S]	DISTRIBUTION 1	DISTRIBUTION 2	DISTRIBUTION 3	DISTRIBUTION 4
[1, 1]	0.089784904	0.546740971	1.158606735	1.251692994
[1, 2]	0.069779892	0.333874080	0.661052509	0.709137327
[1, 3]	0.054587626	0.176850256	0.293924538	0.308754394
[1, 4]	0.045369386	0.095264814	0.107065610	0.105344384
[1, 5]	0.039899065	0.062063708	0.036473439	0.029047134
[1, 6]	0.036009246	0.049474850	0.015536966	0.007051680
[1, 7]	0.032609487	0.043314515	0.010109191	0.002018482
[1, 8]	0.029326525	0.038691220	0.008392391	0.001044391
[1, 9]	0.026067430	0.034377253	0.007419629	0.000883829
[1,10]	0.022823113	0.030231161	0.006836555	0.001147775
[1,11]	0.019626921	0.026620800	0.007489636	0.002757473
[1,12]	0.016578023	0.024684489	0.012029274	0.008601078
[1,13]	0.013845035	0.026416503	0.025138439	0.023786214
[1,14]	0.011489971	0.032664466	0.048882125	0.050573651
[1,15]	0.009052051	0.038160367	0.071008714	0.075614374
[1,16]	0.005448739	0.030063499	0.061341253	0.065992205
[2, 2]	0.118000572	0.492708368	0.963308784	1.032913305
[2, 3]	0.091862634	0.261207621	0.444006890	0.469007245
[2, 4]	0.075875993	0.135252351	0.163062976	0.163989312
[2, 5]	0.066486809	0.083400109	0.053562514	0.045668028
[2, 6]	0.059924945	0.064353666	0.020682779	0.010868101
[2, 7]	0.054247848	0.055758596	0.012336502	0.002825351
[2, 8]	0.048782828	0.049694676	0.009948828	0.001283236
[2, 9]	0.043360917	0.044132771	0.008734852	0.001020662
[2,10]	0.037963755	0.038785308	0.007973298	0.001245691
[2,11]	0.032645378	0.034068725	0.008496560	0.002849570
[2,12]	0.027568766	0.031367496	0.013200827	0.008950551
[2,13]	0.023013185	0.033207768	0.027510467	0.025406870
[2,14]	0.019086127	0.040867569	0.054430936	0.055497020
[2,15]	0.015031762	0.047992222	0.080728869	0.084980661
[3, 3]	0.117027983	0.261377320	0.434970736	0.460121626
[3, 4]	0.096524454	0.132047319	0.165592585	0.169799753
[3, 5]	0.084441204	0.076635443	0.053017377	0.048582580
[3, 6]	0.076050738	0.056601578	0.018095805	0.011440994
[3, 7]	0.068830788	0.048292909	0.009323074	0.002661189
[3, 8]	0.061893697	0.042891541	0.007091012	0.000982438
[3, 9]	0.055014319	0.038067758	0.006146643	0.000700307
[3,10]	0.048167339	0.033446475	0.005559732	0.000794553
[3,11]	0.041421860	0.029350713	0.005765878	0.001703366
[3,12]	0.034985754	0.026933959	0.008621021	0.005353517
[3,13]	0.029212682	0.028339397	0.017825208	0.015628249
[3,14]	0.024234269	0.034737022	0.035873119	0.035219577
[4, 4]	0.115901578	0.114165918	0.133894327	0.138305441
[4, 5]	0.101555256	0.064729788	0.043534888	0.042107656
[4, 6]	0.091494787	0.046371503	0.013339073	0.010064480
[4, 7]	0.082812757	0.039048748	0.005615350	0.002137774
[4, 8]	0.074466915	0.034569057	0.003827394	0.000600464
[4, 9]	0.066190364	0.030666238	0.003235526	0.000349584
[4,10]	0.057953842	0.026953461	0.002904758	0.000362316
[4,11]	0.049842786	0.023692446	0.002960457	0.000719478
[4,12]	0.042111087	0.021833935	0.004294440	0.002240506
[4,13]	0.035185195	0.023097800	0.008773672	0.006713432
[5, 5]	0.121292757	0.058785756	0.031674303	0.031167007
[5, 6]	0.109387950	0.042723544	0.009413833	0.007923196
[5, 7]	0.099030592	0.036025474	0.003302701	0.001625652
[5, 8]	0.089054035	0.031886293	0.001929637	0.000350575
[5, 9]	0.079156922	0.028287335	0.001564198	0.000144669
[5,10]	0.069308019	0.024879866	0.001400979	0.000131037
[5,11]	0.059611870	0.021934183	0.001443461	0.000241323
[5,12]	0.050375677	0.020382369	0.002115490	0.000736075
[6, 6]	0.129386507	0.044872327	0.006618778	0.005587863
[6, 7]	0.117165914	0.038360060	0.002254927	0.001199584
[6, 8]	0.105367498	0.034040859	0.001211387	0.000223586
[6, 9]	0.093658166	0.030212333	0.000952245	0.000061073
[6,10]	0.082005568	0.026585307	0.000859526	0.000045013
[6,11]	0.070534988	0.023479991	0.000926226	0.000081018
[7, 7]	0.136211615	0.043241898	0.001818081	0.000825966
[7, 8]	0.122500478	0.038492433	0.001055052	0.000157156
[7, 9]	0.108887926	0.034180477	0.000843437	0.000035359
[7,10]	0.095340747	0.030083061	0.000770613	0.000023218
[8, 8]	0.139905811	0.043709509	0.001091855	0.000117987
[8, 9]	0.124359846	0.038830828	0.000911058	0.000039128

[R, S]	DISTRIBUTION 5	DISTRIBUTION 6	DISTRIBUTION 7	DISTRIBUTION 8
[1, 1]	1.262244101	0.000920282	5.119236705	2.660965993
[1, 2]	0.714557535	0.001381124	0.709605665	0.369126871
[1, 3]	0.310382253	0.001811602	0.081781153	0.044262137
[1, 4]	0.105086690	0.002136718	0.021068454	0.018160855
[1, 5]	0.028142487	0.002300364	0.015680050	0.028958989
[1, 6]	0.006030756	0.002300866	0.014176805	0.051695495
[1, 7]	0.001047321	0.002178409	0.012880010	0.078569608
[1, 8]	0.000162765	0.001977659	0.011591752	0.096308612
[1, 9]	0.000099673	0.001722842	0.010303774	0.093910127
[1,10]	0.000465055	0.001418947	0.009015831	0.072500801
[1,11]	0.002188666	0.001071861	0.007728612	0.044228226
[1,12]	0.008185747	0.000710743	0.006456011	0.021484833
[1,13]	0.023612332	0.000390839	0.005404270	0.008824633
[1,14]	0.050751573	0.000165474	0.006721511	0.004881541
[1,15]	0.076129441	0.000048556	0.024117780	0.012739595
[1,16]	0.066517467	0.000008319	0.087939598	0.045736491
[2, 2]	1.040767383	0.007855935	0.597674491	0.311032844
[2, 3]	0.471790958	0.010725697	0.065114528	0.034552919
[2, 4]	0.164035484	0.012733174	0.009067727	0.006647185
[2, 5]	0.044713482	0.013656961	0.004644217	0.007531298
[2, 6]	0.009699187	0.013592110	0.004056006	0.013095016
[2, 7]	0.001695570	0.012828194	0.003678900	0.020370255
[2, 8]	0.000254393	0.011636441	0.003310736	0.025946178
[2, 9]	0.000104819	0.010148290	0.002942870	0.026408781
[2,10]	0.000446818	0.008377134	0.002575018	0.021281828
[2,11]	0.002178228	0.006343106	0.002207343	0.013511641
[2,12]	0.008442331	0.004211801	0.001843322	0.006779032
[2,13]	0.025146416	0.002314784	0.001535869	0.002815622
[2,14]	0.055600103	0.000977195	0.001850715	0.001461554
[2,15]	0.085449622	0.000285297	0.006503626	0.003523444
[3, 3]	0.462950012	0.032496986	0.043710079	0.025158093
[3, 4]	0.170260671	0.040240871	0.004636128	0.005644702
[3, 5]	0.048064923	0.043951442	0.001376712	0.005048153
[3, 6]	0.010672505	0.044087228	0.001095549	0.006455388
[3, 7]	0.001893029	0.041757906	0.000988753	0.007998272
[3, 8]	0.000278261	0.037973480	0.000889635	0.008753655
[3, 9]	0.000072475	0.033231195	0.000790781	0.008078415
[3,10]	0.000245128	0.027582687	0.000691936	0.006126233
[3,11]	0.001234445	0.021047321	0.000593151	0.003764039
[3,12]	0.004974583	0.014104658	0.000495540	0.001876157
[3,13]	0.015368595	0.007825467	0.000415257	0.000808887
[3,14]	0.035130789	0.003330903	0.000517916	0.000517583
[4, 4]	0.138830525	0.085264532	0.003260344	0.013655623
[4, 5]	0.041977875	0.095961364	0.001030142	0.014752206
[4, 6]	0.009724479	0.097567799	0.000817665	0.015722195
[4, 7]	0.001772237	0.092979660	0.000737508	0.015637641
[4, 8]	0.000260510	0.084837647	0.000663554	0.014003407
[4, 9]	0.000045384	0.074502141	0.000589820	0.010884547
[4,10]	0.000094006	0.062174937	0.000516095	0.007156832
[4,11]	0.000481723	0.047828866	0.000442429	0.003941049
[4,12]	0.002018635	0.032387509	0.000369878	0.001856575
[4,13]	0.006482027	0.018179046	0.000313080	0.000856220
[5, 5]	0.031173897	0.160169369	0.001143143	0.042203915
[5, 6]	0.007817909	0.165825996	0.000961508	0.045650640
[5, 7]	0.001494328	0.159281215	0.000869669	0.042897527
[5, 8]	0.000224611	0.145880914	0.000782537	0.035775312
[5, 9]	0.000030870	0.128502550	0.000695584	0.026018929
[5,10]	0.000028070	0.107725347	0.000608639	0.016172700
[5,11]	0.000139156	0.083453268	0.000521767	0.008535323
[5,12]	0.000604636	0.057053583	0.000436255	0.003941342
[6, 6]	0.005556257	0.234813819	0.001147387	0.105892510
[6, 7]	0.001158956	0.227680688	0.001040046	0.103529555
[6, 8]	0.000182944	0.209373699	0.000935916	0.085185165
[6, 9]	0.000023438	0.184908251	0.000831922	0.060362195
[6,10]	0.000007860	0.155532053	0.000727935	0.036544315
[6,11]	0.000031249	0.121135173	0.000624036	0.018841039
[7, 7]	0.000808034	0.288752541	0.001213140	0.196464860
[7, 8]	0.000140785	0.266671371	0.001091740	0.169715119
[7, 9]	0.000019663	0.236024440	0.000970433	0.120458498
[7,10]	0.000004581	0.198982240	0.000849133	0.072083330
[8, 8]	0.000105680	0.315132682	0.001247695	0.268086900
[8, 9]	0.000027468	0.279493498	0.001109060	0.200225288

[R, S]	DISTRIBUTION 9	DISTRIBUTION 10	DISTRIBUTION 11
[1, 1]	1.332819820	0.354638416	0.144260252
[1, 2]	0.185101996	0.049852152	0.022554903
[1, 3]	0.023344427	0.008664491	0.009540711
[1, 4]	0.013461384	0.010461929	0.015184644
[1, 5]	0.026126398	0.021004101	0.023130904
[1, 6]	0.049104997	0.035093229	0.029219475
[1, 7]	0.075251368	0.047845837	0.031863926
[1, 8]	0.092055538	0.054121508	0.031264630
[1, 9]	0.089636874	0.051392001	0.028379861
[1,10]	0.069309646	0.041021665	0.023979087
[1,11]	0.042333818	0.027231296	0.018503921
[1,12]	0.020318100	0.014699590	0.012513440
[1,13]	0.007804716	0.006266386	0.006997146
[1,14]	0.003289865	0.002230157	0.003068296
[1,15]	0.006516472	0.002018574	0.001506720
[1,16]	0.022927086	0.006150794	0.002675578
[2, 2]	0.156210848	0.042800498	0.022282190
[2, 3]	0.018047735	0.006855112	0.009932195
[2, 4]	0.004862778	0.004780789	0.011461282
[2, 5]	0.007049958	0.007457014	0.014247510
[2, 6]	0.012755448	0.011275322	0.016200027
[2, 7]	0.019844199	0.014836452	0.016784562
[2, 8]	0.025095079	0.016671758	0.016052302
[2, 9]	0.025411133	0.015926239	0.014312976
[2,10]	0.020453605	0.012863264	0.011855796
[2,11]	0.012985204	0.008669005	0.008925396
[2,12]	0.006450608	0.004762460	0.005871178
[2,13]	0.002534477	0.002069899	0.003198466
[2,14]	0.001032396	0.000752551	0.001391004
[2,15]	0.001857937	0.000680681	0.000776078
[3, 3]	0.015584538	0.011356953	0.024407195
[3, 4]	0.006966948	0.011988465	0.029779349
[3, 5]	0.007650408	0.014720807	0.033613879
[3, 6]	0.009126920	0.016658017	0.034765191
[3, 7]	0.010304319	0.017005155	0.033600384
[3, 8]	0.010458078	0.015504012	0.030742713
[3, 9]	0.009153634	0.012482266	0.026623507
[3,10]	0.006702006	0.008748478	0.021468974
[3,11]	0.004014723	0.005237235	0.015619399
[3,12]	0.001942301	0.002614218	0.009815629
[3,13]	0.000781733	0.001070911	0.005058500
[3,14]	0.000416448	0.000452607	0.002111771
[4, 4]	0.021482581	0.034812101	0.066996430
[4, 5]	0.026069427	0.044814000	0.077959861
[4, 6]	0.028079506	0.050704966	0.081396224
[4, 7]	0.026980456	0.050805025	0.078830080
[4, 8]	0.022981637	0.044972494	0.072185861
[4, 9]	0.017023188	0.034875633	0.062675334
[4,10]	0.010735195	0.023383176	0.050793179
[4,11]	0.005672366	0.013312209	0.037193192
[4,12]	0.002516324	0.006298134	0.023520131
[4,13]	0.001025045	0.002460062	0.012170916
[5, 5]	0.071874301	0.102535218	0.138325480
[5, 6]	0.082158852	0.121526734	0.147730357
[5, 7]	0.078990933	0.125205117	0.144594401
[5, 8]	0.065486819	0.112993129	0.133188947
[5, 9]	0.046729059	0.088858200	0.116270156
[5,10]	0.028350711	0.060130683	0.094917660
[5,11]	0.014459400	0.034384396	0.070190588
[5,12]	0.006241045	0.016257095	0.044910983
[6, 6]	0.180863230	0.224844547	0.219583209
[6, 7]	0.186188667	0.241466316	0.217812758
[6, 8]	0.157796937	0.224263402	0.201964290
[6, 9]	0.112599462	0.180401581	0.177180299
[6,10]	0.067664600	0.124338083	0.145526876
[6,11]	0.034048757	0.072097343	0.108531166
[7, 7]	0.335047592	0.374439775	0.285545551
[7, 8]	0.304124967	0.360734052	0.266675284
[7, 9]	0.223083727	0.298316787	0.234968962
[7,10]	0.135010745	0.210396709	0.193887184
[8, 8]	0.456450492	0.481164253	0.320622331
[8, 9]	0.358196483	0.411640050	0.283709111

COVARIANCE[X[R],X[S]] FOR SAMPLE SIZE 32

[R, S]	DISTRIBUTION 1	DISTRIBUTION 2	DISTRIBUTION 3	DISTRIBUTION 4
[1, 1]	0.032941297	0.246235278	0.543922646	0.589983954
[1, 2]	0.029937837	0.211935845	0.462320615	0.500840516
[1, 3]	0.026009343	0.164443129	0.348292931	0.376166185
[1, 4]	0.021928533	0.114808298	0.228905895	0.245607703
[1, 5]	0.018543917	0.074067505	0.131109051	0.138680846
[1, 6]	0.016066573	0.046802637	0.066290138	0.067869705
[1, 7]	0.014521207	0.031500978	0.030647932	0.029010196
[1, 8]	0.013511997	0.023993286	0.014055460	0.011011108
[1, 9]	0.012775103	0.020528490	0.007377152	0.003871986
[1,10]	0.012182206	0.018808723	0.004972021	0.001415232
[1,11]	0.011644709	0.017738913	0.004130716	0.000668862
[1,12]	0.011111632	0.016874450	0.003781715	0.000459599
[1,13]	0.010573441	0.016052574	0.003563782	0.000396437
[1,14]	0.010041193	0.015237506	0.003375643	0.000367962
[1,15]	0.009518796	0.014436761	0.003196605	0.000347076
[1,16]	0.008994851	0.013643730	0.003020899	0.000327784
[1,17]	0.008461413	0.012840688	0.002843405	0.000308524
[1,18]	0.007927171	0.012030120	0.002664072	0.000289229
[1,19]	0.007401422	0.011227983	0.002487061	0.000270942
[1,20]	0.006877891	0.010434449	0.002315485	0.000256859
[1,21]	0.006347875	0.009641048	0.002158104	0.000259441
[1,22]	0.005817315	0.008865033	0.002055314	0.000323120
[1,23]	0.005297517	0.008166318	0.002129652	0.000579800
[1,24]	0.004793682	0.007666072	0.002678054	0.001354467
[1,25]	0.004318187	0.007598809	0.004287080	0.003290730
[1,26]	0.003900351	0.008319960	0.007818137	0.007331405
[1,27]	0.003563842	0.010120360	0.013972179	0.014243187
[1,28]	0.003291846	0.012812145	0.022299677	0.023534653
[1,29]	0.003006338	0.015342136	0.030241098	0.032404275
[1,30]	0.002582061	0.015978867	0.033575567	0.036230801
[1,31]	0.001918917	0.013354468	0.028967709	0.031362272
[1,32]	0.001024295	0.007566347	0.016651260	0.018054175
[2, 2]	0.056631485	0.388366727	0.843146509	0.913011144
[2, 3]	0.049261101	0.304318668	0.644122326	0.695708480
[2, 4]	0.041515563	0.213393885	0.427342385	0.458856037
[2, 5]	0.034947279	0.137404166	0.246193464	0.260928577
[2, 6]	0.030330790	0.086152639	0.124761595	0.128325289
[2, 7]	0.027170916	0.057145371	0.057506550	0.055018849
[2, 8]	0.025383440	0.042982145	0.026058585	0.020895654
[2, 9]	0.023977638	0.036479071	0.013376319	0.007314616
[2,10]	0.022788203	0.033280611	0.008814759	0.002630512
[2,11]	0.021812492	0.031351813	0.007234056	0.001206600
[2,12]	0.020851662	0.029826148	0.006596448	0.000809215
[2,13]	0.019817163	0.028368576	0.006211054	0.000691692
[2,14]	0.018794349	0.026919431	0.005881495	0.000640524
[2,15]	0.017832405	0.025505752	0.005568302	0.000603770
[2,16]	0.016870441	0.024110599	0.005262291	0.000570162
[2,17]	0.015861061	0.022691495	0.004953826	0.000536734
[2,18]	0.014843948	0.021254717	0.004641385	0.000503140
[2,19]	0.013863707	0.019836927	0.004332314	0.000471075
[2,20]	0.012895249	0.018437909	0.004032603	0.000445736
[2,21]	0.011899958	0.017035547	0.003755990	0.000447198
[2,22]	0.010896488	0.015658640	0.003568016	0.000548072
[2,23]	0.009923125	0.014414573	0.003672533	0.000967137
[2,24]	0.008984348	0.013511081	0.004569951	0.002250642
[2,25]	0.008090918	0.013351244	0.007261414	0.005495366
[2,26]	0.007302039	0.014561542	0.013241823	0.012335287
[2,27]	0.006669834	0.017672470	0.023771187	0.024147086
[2,28]	0.006160024	0.022385091	0.038158680	0.040177919
[2,29]	0.005624759	0.026871704	0.052038388	0.055657251
[2,30]	0.004832290	0.028073057	0.058055115	0.062543728
[2,31]	0.003593406	0.023526428	0.050273626	0.054346445
[3, 3]	0.067224003	0.390853769	0.820723774	0.885900178
[3, 4]	0.056655746	0.277576476	0.556794473	0.598212435
[3, 5]	0.047575139	0.178918154	0.325301034	0.345686087
[3, 6]	0.041019915	0.110845488	0.165968000	0.171900952
[3, 7]	0.036872456	0.071996084	0.076307596	0.074232655
[3, 8]	0.034005898	0.052776260	0.033943869	0.028268223

[R, S]	DISTRIBUTION 1 (CONT.)	DISTRIBUTION 2 (CONT.)	DISTRIBUTION 3 (CONT.)	DISTRIBUTION 4 (CONT.)
[3, 9]	0.032416022	0.044202442	0.016771291	0.009835592
[3,10]	0.030724611	0.040039020	0.010603556	0.003446401
[3,11]	0.029263324	0.037578568	0.008495629	0.001500167
[3,12]	0.028076241	0.035751295	0.007681863	0.000960179
[3,13]	0.026745361	0.034023635	0.007219887	0.000805632
[3,14]	0.025288663	0.032264459	0.006834239	0.000742641
[3,15]	0.023957205	0.030553902	0.006467902	0.000699189
[3,16]	0.022717167	0.028896179	0.006111792	0.000660059
[3,17]	0.021386339	0.027207220	0.005754758	0.000621457
[3,18]	0.019979802	0.025475511	0.005392225	0.000582585
[3,19]	0.018636732	0.023766834	0.005032089	0.000545111
[3,20]	0.017357613	0.022096056	0.004682535	0.000514548
[3,21]	0.016036089	0.020420798	0.004357726	0.000511797
[3,22]	0.014668747	0.018761400	0.004126123	0.000613685
[3,23]	0.013345071	0.017251968	0.004208854	0.001056321
[3,24]	0.012090859	0.016139281	0.005159133	0.002440213
[3,25]	0.010893087	0.015885067	0.008103216	0.005994558
[3,26]	0.009820605	0.017230484	0.014762476	0.013594968
[3,27]	0.008962642	0.020838304	0.026659812	0.026901088
[3,28]	0.008277377	0.026407202	0.043148940	0.045214315
[3,29]	0.007557664	0.031804159	0.059327822	0.063199183
[3,30]	0.006493931	0.033367997	0.066664724	0.071558508
[4, 4]	0.066773651	0.295243857	0.585457268	0.628604697
[4, 5]	0.055993387	0.192284329	0.352303133	0.375210851
[4, 6]	0.048134043	0.117987413	0.182669906	0.190666021
[4, 7]	0.043012254	0.074515170	0.084135291	0.083517258
[4, 8]	0.039873738	0.052922623	0.036662217	0.032032021
[4, 9]	0.037478677	0.043140083	0.017183592	0.011090797
[4,10]	0.035998514	0.038793352	0.010183539	0.003762834
[4,11]	0.034097638	0.036211706	0.007832700	0.001520136
[4,12]	0.032548103	0.034357586	0.006972749	0.000901058
[4,13]	0.031193923	0.032747095	0.006528468	0.000730689
[4,14]	0.029541456	0.031068289	0.006176644	0.000667829
[4,15]	0.027854702	0.029378696	0.005843132	0.000627557
[4,16]	0.026396202	0.027778447	0.005519652	0.000592070
[4,17]	0.024940244	0.026184306	0.005198232	0.000557465
[4,18]	0.023312598	0.024521933	0.004871817	0.000522670
[4,19]	0.021683598	0.022856591	0.004545501	0.000488777
[4,20]	0.020184505	0.021246179	0.004228045	0.000460207
[4,21]	0.018690284	0.019648482	0.003931402	0.000453530
[4,22]	0.017105703	0.018050084	0.003709687	0.000530445
[4,23]	0.015534865	0.016575860	0.003746007	0.000884866
[4,24]	0.014069745	0.015474246	0.004510011	0.002020842
[4,25]	0.012691409	0.015173017	0.006977323	0.004994131
[4,26]	0.011440957	0.016364285	0.012679989	0.011462531
[4,27]	0.010430034	0.019707319	0.023050210	0.022978368
[4,28]	0.009630869	0.024974453	0.037674449	0.039105055
[4,29]	0.008795574	0.030177166	0.052324732	0.055279755
[5, 5]	0.061610255	0.180206345	0.324630913	0.345683955
[5, 6]	0.052922597	0.110814085	0.174224527	0.182969708
[5, 7]	0.047235785	0.068314750	0.081283515	0.082281258
[5, 8]	0.043622348	0.046692543	0.034860287	0.032040993
[5, 9]	0.041250375	0.036993501	0.015399893	0.011098728
[5,10]	0.039109690	0.032618512	0.008338133	0.003647711
[5,11]	0.037588687	0.030432709	0.006009966	0.001345690
[5,12]	0.035606069	0.028740843	0.005208482	0.000711694
[5,13]	0.033995929	0.027350900	0.004838892	0.000543715
[5,14]	0.032427545	0.026016157	0.004573114	0.000488900
[5,15]	0.030576457	0.024594333	0.004325239	0.000457927
[5,16]	0.028823776	0.023210118	0.004083624	0.000431643
[5,17]	0.027254809	0.021890011	0.003845784	0.000406337
[5,18]	0.025576051	0.020529339	0.003605615	0.000381038
[5,19]	0.023772253	0.019125362	0.003363877	0.000356192
[5,20]	0.022060743	0.017758093	0.003127413	0.000334585
[5,21]	0.020437961	0.016429086	0.002905554	0.000326791
[5,22]	0.018749880	0.015103498	0.002733184	0.000372558
[5,23]	0.017021815	0.013859114	0.002732849	0.000599613
[5,24]	0.015389528	0.012914105	0.003228609	0.001347838

[R, S]	DISTRIBUTION 1 (CONT.)	DISTRIBUTION 2 (CONT.)	DISTRIBUTION 3 (CONT.)	DISTRIBUTION 4 (CONT.)
[5,25]	0.013887136	0.012631106	0.004908266	0.003347171
[5,26]	0.012534192	0.013570923	0.008882025	0.007780194
[5,27]	0.011424192	0.016286370	0.016246680	0.015821915
[5,28]	0.010543302	0.020626267	0.026829782	0.027307045
[6, 6]	0.057233493	0.095801140	0.145927074	0.153456058
[6, 7]	0.051140484	0.058717205	0.070528421	0.072431821
[6, 8]	0.047257104	0.039005816	0.030228997	0.029038955
[6, 9]	0.044588214	0.029988205	0.012662207	0.010168452
[6,10]	0.042518363	0.026065393	0.006159819	0.003267327
[6,11]	0.040419787	0.024043355	0.004021885	0.001097801
[6,12]	0.038796976	0.022800290	0.003334557	0.000498040
[6,13]	0.036745493	0.021599343	0.003054326	0.000343937
[6,14]	0.034976224	0.020530622	0.002877292	0.000299597
[6,15]	0.033201293	0.019467676	0.002721454	0.000278806
[6,16]	0.031258777	0.018354238	0.002568546	0.000262464
[6,17]	0.029422175	0.017277041	0.002417871	0.000246972
[6,18]	0.027657159	0.016221014	0.002267514	0.000231607
[6,19]	0.025790953	0.015130960	0.002116019	0.000216462
[6,20]	0.023897846	0.014035815	0.001966481	0.000202930
[6,21]	0.022083379	0.012973361	0.001825449	0.000196610
[6,22]	0.020282042	0.011935864	0.001713067	0.000218728
[6,23]	0.018448207	0.010961408	0.001699210	0.000338859
[6,24]	0.016665905	0.010211544	0.001973877	0.000746513
[6,25]	0.015019966	0.009986392	0.002948897	0.001859227
[6,26]	0.013567424	0.010731696	0.005305232	0.004375515
[6,27]	0.012380182	0.012872541	0.009748485	0.009031352
[7, 7]	0.055833426	0.049621607	0.055181279	0.056914225
[7, 8]	0.051673666	0.032973848	0.024361032	0.024100792
[7, 9]	0.048795839	0.025041523	0.009916002	0.008681271
[7,10]	0.046463249	0.021529987	0.004352578	0.002775428
[7,11]	0.044377279	0.019819302	0.002503187	0.000863944
[7,12]	0.042224865	0.018669645	0.001928720	0.000327920
[7,13]	0.040420801	0.017796410	0.001727811	0.000192598
[7,14]	0.038211215	0.016837098	0.001616641	0.000157658
[7,15]	0.036237194	0.015960375	0.001527417	0.000144657
[7,16]	0.034297242	0.015096126	0.001442341	0.000135853
[7,17]	0.032230172	0.014192971	0.001357147	0.000127760
[7,18]	0.030196674	0.013300284	0.001272289	0.000119792
[7,19]	0.028212773	0.012422496	0.001187737	0.000111947
[7,20]	0.026203671	0.011538455	0.001103979	0.000104798
[7,21]	0.024177092	0.010655203	0.001024288	0.000100859
[7,22]	0.022166813	0.009796444	0.000960123	0.000109807
[7,23]	0.020187877	0.009010664	0.000948949	0.000163865
[7,24]	0.018265546	0.008417838	0.001092432	0.000352633
[7,25]	0.016453369	0.008260815	0.001613338	0.000878337
[7,26]	0.014853534	0.008917323	0.002885069	0.002089857
[8, 8]	0.057110473	0.029434428	0.018196959	0.018137976
[8, 9]	0.053967976	0.022720979	0.007534295	0.006927703
[8,10]	0.051411943	0.019613464	0.003105751	0.002260941
[8,11]	0.049055014	0.018046506	0.001578659	0.000674844
[8,12]	0.046829907	0.017038354	0.001108985	0.000217345
[8,13]	0.044535474	0.016161131	0.000957708	0.000101608
[8,14]	0.042448089	0.015386784	0.000889755	0.000073886
[8,15]	0.040042145	0.014521340	0.000837697	0.000065706
[8,16]	0.037875158	0.013732323	0.000790925	0.000061354
[8,17]	0.035729966	0.012950428	0.000744921	0.000057653
[8,18]	0.033434429	0.012121637	0.000697940	0.000054042
[8,19]	0.031164084	0.011300842	0.000651222	0.000050494
[8,20]	0.028984946	0.010509832	0.000605708	0.000047233
[8,21]	0.026789078	0.009719198	0.000562527	0.000045263
[8,22]	0.024535343	0.008930056	0.000528089	0.000048522
[8,23]	0.022315171	0.008213143	0.000524389	0.000070245
[8,24]	0.020207892	0.007698706	0.000608815	0.000147718
[8,25]	0.018228438	0.007601989	0.000904146	0.000366598
[9, 9]	0.059863115	0.022376037	0.005506715	0.005070706
[9,10]	0.057043412	0.019622875	0.002295943	0.001755366
[9,11]	0.054446975	0.018145055	0.001100434	0.000525102
[9,12]	0.051941482	0.017141197	0.000721040	0.000152094

[R, S]	DISTRIBUTION 1 (CONT.)	DISTRIBUTION 2 (CONT.)	DISTRIBUTION 3 (CONT.)	DISTRIBUTION 4 (CONT.)
[9,13]	0.049502165	0.016293630	0.000603112	0.000055564
[9,14]	0.046970091	0.015452155	0.000554134	0.000033103
[9,15]	0.044577072	0.014661759	0.000521972	0.000027586
[9,16]	0.041987774	0.013811790	0.000491595	0.000025408
[9,17]	0.039588587	0.013021393	0.000463104	0.000023827
[9,18]	0.037157183	0.012220963	0.000434463	0.000022333
[9,19]	0.034603363	0.011382660	0.000405064	0.000020863
[9,20]	0.032115247	0.010566264	0.000376578	0.000019518
[9,21]	0.029708152	0.009780897	0.000350532	0.000018716
[9,22]	0.027252091	0.009001082	0.000331068	0.000020053
[9,23]	0.024764526	0.008276708	0.000333710	0.000028851
[9,24]	0.022389920	0.007762216	0.000398859	0.000060016
[10,10]	0.063041621	0.020701780	0.001735368	0.001261642
[10,11]	0.060190844	0.019270945	0.000871665	0.000398931
[10,12]	0.057433524	0.018239590	0.000574313	0.000112430
[10,13]	0.054711359	0.017337227	0.000478055	0.000034570
[10,14]	0.051985243	0.016465015	0.000438536	0.000016274
[10,15]	0.049194602	0.015579832	0.000411994	0.000012149
[10,16]	0.046530928	0.014735626	0.000389060	0.000010919
[10,17]	0.043727786	0.013848245	0.000365645	0.000010188
[10,18]	0.041035083	0.012995348	0.000343082	0.000009546
[10,19]	0.038312149	0.012133339	0.000320405	0.000008926
[10,20]	0.035527814	0.011253743	0.000297739	0.000008369
[10,21]	0.032801311	0.010398465	0.000277279	0.000008109
[10,22]	0.030118511	0.009579521	0.000263676	0.000008968
[10,23]	0.027416064	0.008825487	0.000270620	0.000013550
[11,11]	0.066105933	0.020862658	0.000755472	0.000283796
[11,12]	0.063082400	0.019780930	0.000535957	0.000084358
[11,13]	0.060089600	0.018808914	0.000456566	0.000024854
[11,14]	0.057083169	0.017860333	0.000420829	0.000010201
[11,15]	0.054063267	0.016914011	0.000395954	0.000006922
[11,16]	0.051041420	0.015968387	0.000373371	0.000006053
[11,17]	0.048103252	0.015049067	0.000351788	0.000005629
[11,18]	0.045033699	0.014088921	0.000329376	0.000005265
[11,19]	0.042051337	0.013156287	0.000307674	0.000004928
[11,20]	0.039076172	0.012227561	0.000286482	0.000004649
[11,21]	0.036048965	0.011289521	0.000266831	0.000004599
[11,22]	0.033048951	0.010384837	0.000254248	0.000005409
[12,12]	0.068813224	0.021495836	0.000538490	0.000060666
[12,13]	0.065532603	0.020441796	0.000473992	0.000019380
[12,14]	0.062245130	0.019409573	0.000440122	0.000008172
[12,15]	0.058951987	0.018381353	0.000414685	0.000005525
[12,16]	0.055672576	0.017358598	0.000391227	0.000004813
[12,17]	0.052407597	0.016340561	0.000368225	0.000004467
[12,18]	0.049165200	0.015329661	0.000345456	0.000004184
[12,19]	0.045823628	0.014288258	0.000322111	0.000003914
[12,20]	0.042582987	0.013280108	0.000299964	0.000003708
[12,21]	0.039350151	0.012281899	0.000279925	0.000003735
[13,13]	0.070985533	0.022122957	0.000503813	0.000015264
[13,14]	0.067418606	0.021005327	0.000471208	0.000007437
[13,15]	0.063852832	0.019893138	0.000444596	0.000005355
[13,16]	0.060308662	0.018788752	0.000419597	0.000004731
[13,17]	0.056767326	0.017685455	0.000394910	0.000004400
[13,18]	0.053223014	0.016581340	0.000370272	0.000004121
[13,19]	0.049684903	0.015479575	0.000345797	0.000003862
[13,20]	0.046087236	0.014361226	0.000321446	0.000003658
[14,14]	0.072575099	0.022607679	0.000505419	0.000007059
[14,15]	0.068754556	0.021416459	0.000477586	0.000005525
[14,16]	0.064943677	0.020229209	0.000450862	0.000004970
[14,17]	0.061141445	0.019044843	0.000424427	0.000004636
[14,18]	0.057313302	0.017852518	0.000397874	0.000004343
[14,19]	0.053488207	0.016661595	0.000371471	0.000004069
[15,15]	0.073665294	0.022945307	0.000511381	0.000005779
[15,16]	0.069594181	0.021677059	0.000482935	0.000005288
[15,17]	0.065513207	0.020405914	0.000454587	0.000004946
[15,18]	0.061423427	0.019132142	0.000426233	0.000004636
[16,16]	0.074260366	0.023130340	0.000515269	0.000005626
[16,17]	0.069891730	0.021769607	0.000484938	0.000005275

[R, S]	DISTRIBUTION 5	DISTRIBUTION 6	DISTRIBUTION 7	DISTRIBUTION 8
[1, 1]	0.595218648	0.000004687	6.982283197	3.629301111
[1, 2]	0.505214268	0.000006483	1.859317118	0.966449172
[1, 3]	0.379323830	0.000009949	0.368650211	0.191619904
[1, 4]	0.247489128	0.000016888	0.067158846	0.034910658
[1, 5]	0.139519399	0.000029136	0.020521025	0.010682099
[1, 6]	0.068024088	0.000047213	0.014378730	0.007553311
[1, 7]	0.028797720	0.000069576	0.013303784	0.007242177
[1, 8]	0.010638759	0.000092651	0.012746450	0.007738873
[1, 9]	0.003447904	0.000112619	0.012233317	0.009553384
[1,10]	0.000986236	0.000126754	0.011723393	0.013927578
[1,11]	0.000251698	0.000134291	0.011213669	0.022421279
[1,12]	0.000059365	0.000136131	0.010703957	0.036163297
[1,13]	0.000014865	0.000133912	0.010194244	0.054732056
[1,14]	0.000005633	0.000129233	0.009684532	0.075305836
[1,15]	0.000003805	0.000123263	0.009174820	0.092952892
[1,16]	0.000003354	0.000116681	0.008665108	0.102391990
[1,17]	0.000003152	0.000109779	0.008155395	0.100452299
[1,18]	0.000003136	0.000102594	0.007645683	0.087708694
[1,19]	0.000003973	0.000094967	0.007135971	0.068171195
[1,20]	0.000008860	0.000086588	0.006626259	0.047251024
[1,21]	0.000030699	0.000077067	0.006116547	0.029374859
[1,22]	0.000114387	0.000066103	0.005606834	0.016635857
[1,23]	0.000392887	0.000053747	0.005097124	0.008907166
[1,24]	0.001194388	0.000040627	0.004587436	0.004849279
[1,25]	0.003169029	0.000027941	0.004078070	0.002949347
[1,26]	0.007268930	0.000017087	0.003572566	0.002092257
[1,27]	0.014268298	0.000009075	0.003105593	0.001669407
[1,28]	0.023670924	0.000004100	0.002950587	0.001544084
[1,29]	0.032647624	0.000001568	0.004764629	0.002478112
[1,30]	0.036531376	0.000000536	0.015598246	0.008107929
[1,31]	0.031634033	0.000000193	0.051743929	0.026895840
[1,32]	0.018213563	0.000000093	0.103259671	0.053673052
[2, 2]	0.920942180	0.000041005	1.898333669	0.986729473
[2, 3]	0.701553692	0.000061120	0.398611613	0.207193286
[2, 4]	0.462409604	0.000087922	0.068083112	0.035389478
[2, 5]	0.262566448	0.000124626	0.014731407	0.007662294
[2, 6]	0.128688104	0.000171796	0.007789326	0.004075974
[2, 7]	0.054691613	0.000225253	0.006843991	0.003674307
[2, 8]	0.020264374	0.000277904	0.006524636	0.003806680
[2, 9]	0.006582198	0.000321098	0.006259577	0.004494999
[2,10]	0.001885713	0.000350265	0.005998507	0.006283854
[2,11]	0.000481422	0.000364109	0.005737688	0.009943802
[2,12]	0.000113137	0.000364993	0.005476884	0.016151161
[2,13]	0.000027841	0.000356929	0.005216080	0.024946942
[2,14]	0.000010151	0.000343477	0.004955276	0.035224098
[2,15]	0.000006673	0.000327143	0.004694472	0.044692710
[2,16]	0.000005840	0.000309409	0.004433668	0.050605127
[2,17]	0.000005475	0.000290848	0.004172864	0.050992736
[2,18]	0.000005407	0.000271375	0.003912060	0.045677570
[2,19]	0.000006667	0.000250451	0.003651256	0.036367489
[2,20]	0.000014335	0.000227224	0.003390452	0.025767488
[2,21]	0.000049218	0.000200757	0.003129648	0.016321841
[2,22]	0.000184754	0.000170512	0.002868844	0.009363561
[2,23]	0.000641419	0.000136998	0.002608041	0.005024110
[2,24]	0.001970682	0.000102194	0.002347247	0.002695238
[2,25]	0.005280244	0.000069331	0.002086600	0.001591095
[2,26]	0.012219970	0.000041846	0.001827726	0.001096454
[2,27]	0.024179869	0.000021974	0.001586854	0.000859793
[2,28]	0.040400022	0.000009852	0.001494158	0.000783360
[2,29]	0.056063705	0.000003772	0.002352219	0.001223651
[2,30]	0.063051207	0.000001315	0.007639531	0.003971041
[2,31]	0.054808154	0.000000492	0.025598191	0.013305626
[3, 3]	0.893283819	0.000264770	0.301003068	0.156457695
[3, 4]	0.602888177	0.000382554	0.052834214	0.027462944
[3, 5]	0.347963131	0.000511267	0.008768365	0.004559343
[3, 6]	0.172529771	0.000647938	0.002838985	0.001483356
[3, 7]	0.073948852	0.000782554	0.002138131	0.001142899
[3, 8]	0.027574870	0.000899854	0.002003724	0.001151529

[R, S]	DISTRIBUTION 5 (CONT.)	DISTRIBUTION 6 (CONT.)	DISTRIBUTION 7 (CONT.)	DISTRIBUTION 8 (CONT.)
[3, 9]	0.009000273	0.000987838	0.001919720	0.001325460
[3,10]	0.002587465	0.001037881	0.001839487	0.001795600
[3,11]	0.000661483	0.001053864	0.001759496	0.002779950
[3,12]	0.000154655	0.001041569	0.001679519	0.004490656
[3,13]	0.000037000	0.001010309	0.001599541	0.006984968
[3,14]	0.000012606	0.000968432	0.001519564	0.010004446
[3,15]	0.000007859	0.000920787	0.001439587	0.012927676
[3,16]	0.000006776	0.000869997	0.001359610	0.014939559
[3,17]	0.000006329	0.000817040	0.001279633	0.015381098
[3,18]	0.000006196	0.000761178	0.001199656	0.014083186
[3,19]	0.000007387	0.000700394	0.001119679	0.011459618
[3,20]	0.000015081	0.000632060	0.001039702	0.008291586
[3,21]	0.000050910	0.000553765	0.000959725	0.005353098
[3,22]	0.000192668	0.000464768	0.000879748	0.003116955
[3,23]	0.000678194	0.000367716	0.000799771	0.001682899
[3,24]	0.002113366	0.000269334	0.000719797	0.000894937
[3,25]	0.005739085	0.000179084	0.000639864	0.000515282
[3,26]	0.013448534	0.000105887	0.000560433	0.000345241
[3,27]	0.026916966	0.000054551	0.000486161	0.000265850
[3,28]	0.045439847	0.000024119	0.000454811	0.000238986
[3,29]	0.063632368	0.000009231	0.000702057	0.000365324
[3,30]	0.072110148	0.000003316	0.002263358	0.001176550
[4, 4]	0.633477480	0.001208860	0.036402683	0.018923133
[4, 5]	0.377781811	0.001670357	0.005754951	0.002993939
[4, 6]	0.191539530	0.002113400	0.001211520	0.000635785
[4, 7]	0.083410151	0.002508214	0.000678596	0.000370422
[4, 8]	0.031468813	0.002822501	0.000608103	0.000368223
[4, 9]	0.010362029	0.003032823	0.000580410	0.000441081
[4,10]	0.002998248	0.003137337	0.000556009	0.000618134
[4,11]	0.000769107	0.003145831	0.000531823	0.000960729
[4,12]	0.000178926	0.003087244	0.000507649	0.001519881
[4,13]	0.000041322	0.002982216	0.000483475	0.002293058
[4,14]	0.000012766	0.002851720	0.000459301	0.003185791
[4,15]	0.000007279	0.002708985	0.000435127	0.004009544
[4,16]	0.000006105	0.002558853	0.000410954	0.004536930
[4,17]	0.000005668	0.002402588	0.000386780	0.004598138
[4,18]	0.000005501	0.002237865	0.000362606	0.004165092
[4,19]	0.000006332	0.002058264	0.000338432	0.003367961
[4,20]	0.000012148	0.001855303	0.000314259	0.002431312
[4,21]	0.000039976	0.001621657	0.000290085	0.001571675
[4,22]	0.000152328	0.001355632	0.000265911	0.000919235
[4,23]	0.000544376	0.001066175	0.000241737	0.000499856
[4,24]	0.001724083	0.000774639	0.000217565	0.000268066
[4,25]	0.004756233	0.000509934	0.000193406	0.000155420
[4,26]	0.011312840	0.000298082	0.000169412	0.000104459
[4,27]	0.022959893	0.000151756	0.000147084	0.000080522
[4,28]	0.039258284	0.000066426	0.000138396	0.000072785
[4,29]	0.055607323	0.000025347	0.000216997	0.000113038
[5, 5]	0.348053908	0.004146440	0.003923251	0.002050028
[5, 6]	0.183944901	0.005459688	0.000815600	0.000440652
[5, 7]	0.082376989	0.006585357	0.000413940	0.000244900
[5, 8]	0.031703290	0.007446319	0.000362140	0.000251622
[5, 9]	0.010593114	0.008001057	0.000344825	0.000322218
[5,10]	0.003098323	0.008256523	0.000330269	0.000477437
[5,11]	0.000800117	0.008265629	0.000315899	0.000755328
[5,12]	0.000185676	0.008092628	0.000301540	0.001174910
[5,13]	0.000041374	0.007811591	0.000287181	0.001707304
[5,14]	0.000011332	0.007465404	0.000272821	0.002261252
[5,15]	0.000005626	0.007088767	0.000258462	0.002700720
[5,16]	0.000004489	0.006696684	0.000244103	0.002895729
[5,17]	0.000004125	0.006289737	0.000229744	0.002781985
[5,18]	0.000003971	0.005861589	0.000215385	0.002392921
[5,19]	0.000004423	0.005396642	0.000201026	0.001842933
[5,20]	0.000007931	0.004871942	0.000186667	0.001273021
[5,21]	0.000025214	0.004266421	0.000172308	0.000793215
[5,22]	0.000096466	0.003573757	0.000157949	0.000452772
[5,23]	0.000349974	0.002815875	0.000143590	0.000245513
[5,24]	0.001127407	0.002048502	0.000129232	0.000135604

[R, S]	DISTRIBUTION 5 (CONT.)	DISTRIBUTION 6 (CONT.)	DISTRIBUTION 7 (CONT.)	DISTRIBUTION 8 (CONT.)
[5,25]	0.003163351	0.001348996	0.000114884	0.000083341
[5,26]	0.007648388	0.000787974	0.000100657	0.000059337
[5,27]	0.015766315	0.000400396	0.000087610	0.000047402
[5,28]	0.027352838	0.000174727	0.000083937	0.000044260
[6, 6]	0.154307725	0.011218355	0.000697032	0.000424202
[6, 7]	0.072649462	0.014029732	0.000417693	0.000306403
[6, 8]	0.028906651	0.016160378	0.000374371	0.000329680
[6, 9]	0.009888321	0.017529797	0.000357113	0.000415567
[6,10]	0.002941888	0.018177071	0.000342076	0.000594921
[6,11]	0.000768532	0.018234079	0.000327194	0.000909723
[6,12]	0.000178751	0.017875578	0.000312321	0.001376660
[6,13]	0.000038696	0.017254672	0.000297449	0.001956125
[6,14]	0.000009344	0.016497013	0.000282576	0.002539732
[6,15]	0.000003807	0.015665105	0.000267704	0.002975471
[6,16]	0.000002775	0.014799511	0.000252831	0.003128112
[6,17]	0.000002502	0.013906313	0.000237959	0.002943689
[6,18]	0.000002389	0.012969398	0.000223087	0.002477174
[6,19]	0.000002584	0.011956625	0.000208214	0.001864566
[6,20]	0.000004328	0.010818713	0.000193342	0.001258331
[6,21]	0.000013181	0.009505289	0.000178469	0.000767092
[6,22]	0.000050432	0.007995640	0.000163597	0.000430637
[6,23]	0.000185534	0.006330978	0.000148724	0.000232544
[6,24]	0.000607854	0.004629770	0.000133853	0.000130583
[6,25]	0.001735377	0.003064292	0.000118993	0.000083036
[6,26]	0.004268279	0.001797844	0.000104266	0.000061062
[6,27]	0.008945762	0.000916510	0.000090836	0.000049760
[7, 7]	0.057126442	0.024747733	0.000465193	0.000530566
[7, 8]	0.024089987	0.029347829	0.000428680	0.000616444
[7, 9]	0.008560300	0.032340805	0.000409797	0.000743558
[7,10]	0.002615106	0.033825162	0.000392595	0.000961903
[7,11]	0.000695854	0.034089359	0.000375518	0.001327095
[7,12]	0.000163315	0.033494548	0.000358449	0.001860912
[7,13]	0.000034773	0.032372613	0.000341380	0.002516697
[7,14]	0.000007534	0.030960005	0.000324311	0.003167241
[7,15]	0.000002397	0.029411084	0.000307242	0.003636279
[7,16]	0.000001486	0.027789030	0.000290173	0.003769978
[7,17]	0.000001289	0.026119523	0.000273104	0.003511501
[7,18]	0.000001218	0.024377785	0.000256035	0.002931181
[7,19]	0.000001285	0.022503374	0.000238966	0.002191565
[7,20]	0.000002014	0.020407402	0.000221897	0.001470852
[7,21]	0.000005834	0.017992027	0.000204828	0.000893095
[7,22]	0.000022220	0.015205571	0.000187759	0.000500838
[7,23]	0.000082735	0.012109208	0.000170690	0.000271626
[7,24]	0.000275448	0.008912975	0.000153622	0.000154328
[7,25]	0.000799945	0.005939276	0.000136567	0.000099702
[7,26]	0.002001902	0.003507322	0.000119667	0.000074252
[8, 8]	0.018160673	0.045678508	0.000488338	0.001375626
[8, 9]	0.006889277	0.051438932	0.000467616	0.001701519
[8,10]	0.002194851	0.054438811	0.000448035	0.002037226
[8,11]	0.000600907	0.055214856	0.000428549	0.002477308
[8,12]	0.000143547	0.054432099	0.000409069	0.003069365
[8,13]	0.000030505	0.052691427	0.000389590	0.003769233
[8,14]	0.000006146	0.050432920	0.000370110	0.004433159
[8,15]	0.000001518	0.047919986	0.000350631	0.004861767
[8,16]	0.000000720	0.045290958	0.000331151	0.004886632
[8,17]	0.000000574	0.042578904	0.000311672	0.004455896
[8,18]	0.000000533	0.039761759	0.000292192	0.003665367
[8,19]	0.000000550	0.036746736	0.000272713	0.002713626
[8,20]	0.000000811	0.033390159	0.000253233	0.001810833
[8,21]	0.000002222	0.029531153	0.000233754	0.001098267
[8,22]	0.000008381	0.025069492	0.000214274	0.000619148
[8,23]	0.000031506	0.020078390	0.000194795	0.000340723
[8,24]	0.000106447	0.014877178	0.000175317	0.000198343
[8,25]	0.000314238	0.009985208	0.000155854	0.000131515
[9, 9]	0.005059208	0.072260155	0.000525965	0.003970381
[9,10]	0.001731394	0.077631944	0.000503984	0.004870501
[9,11]	0.000495808	0.079372784	0.000482067	0.005585209
[9,12]	0.000121946	0.078572866	0.000460154	0.006265227

[R, S]	DISTRIBUTION 5 (CONT.)	DISTRIBUTION 6 (CONT.)	DISTRIBUTION 7 (CONT.)	DISTRIBUTION 8 (CONT.)
[9,13]	0.000026246	0.076214778	0.000438242	0.006928550
[9,14]	0.000005116	0.073013643	0.000416330	0.007444411
[9,15]	0.000001040	0.069405427	0.000394418	0.007606243
[9,16]	0.000000341	0.065607263	0.000372506	0.007250428
[9,17]	0.000000226	0.061695907	0.000350594	0.006358688
[9,18]	0.000000203	0.057636400	0.000328682	0.005085122
[9,19]	0.000000205	0.053313712	0.000306769	0.003691716
[9,20]	0.000000286	0.048524132	0.000284857	0.002434867
[9,21]	0.000000738	0.043030890	0.000262945	0.001472668
[9,22]	0.000002743	0.036673114	0.000241033	0.000838124
[9,23]	0.000010379	0.029524123	0.000219121	0.000473414
[9,24]	0.000035518	0.022012549	0.000197210	0.000287136
[10,10]	0.001250878	0.100413870	0.000559976	0.010757521
[10,11]	0.000386761	0.103660458	0.000535626	0.012808036
[10,12]	0.000099695	0.103123937	0.000511279	0.013976306
[10,13]	0.000022065	0.100269379	0.000486932	0.014553663
[10,14]	0.000004297	0.096162212	0.000462586	0.014585126
[10,15]	0.000000787	0.091451395	0.000438239	0.013952900
[10,16]	0.000000179	0.086465960	0.000413893	0.012569658
[10,17]	0.000000084	0.081323235	0.000389546	0.010527561
[10,18]	0.000000068	0.075998916	0.000365199	0.008119905
[10,19]	0.000000067	0.070343609	0.000340853	0.005738059
[10,20]	0.000000089	0.064105444	0.000316506	0.003718660
[10,21]	0.000000217	0.056970086	0.000292159	0.002235232
[10,22]	0.000000792	0.048708523	0.000267813	0.001284088
[10,23]	0.000003004	0.039384817	0.000243466	0.000746767
[11,11]	0.000276389	0.125731051	0.000589188	0.025944592
[11,12]	0.000077250	0.125787810	0.000562407	0.029712041
[11,13]	0.000017937	0.122639306	0.000535625	0.030800478
[11,14]	0.000003577	0.117760047	0.000508844	0.029953649
[11,15]	0.000000635	0.112048324	0.000482063	0.027552175
[11,16]	0.000000114	0.105962867	0.000455282	0.023853957
[11,17]	0.000000033	0.099674919	0.000428500	0.019267519
[11,18]	0.000000021	0.093168706	0.000401719	0.014405098
[11,19]	0.000000020	0.086277074	0.000374938	0.009925360
[11,20]	0.000000025	0.078695196	0.000348157	0.006315058
[11,21]	0.000000059	0.070046291	0.000321375	0.003760438
[11,22]	0.000000210	0.060034902	0.000294594	0.002167272
[12,12]	0.000054756	0.145457980	0.000613535	0.054793329
[12,13]	0.000013834	0.142236612	0.000584319	0.059934362
[12,14]	0.000002894	0.136758485	0.000555103	0.058806258
[12,15]	0.000000521	0.130196704	0.000525887	0.053449037
[12,16]	0.000000085	0.123152216	0.000496671	0.045362713
[12,17]	0.000000016	0.115858150	0.000467455	0.035857459
[12,18]	0.000000007	0.108311092	0.000438239	0.026261667
[12,19]	0.000000005	0.100328784	0.000409023	0.017765049
[12,20]	0.000000007	0.091567218	0.000379807	0.011131323
[12,21]	0.000000017	0.081588532	0.000350591	0.006555798
[13,13]	0.000009740	0.159073378	0.000633012	0.100597047
[13,14]	0.000002224	0.153155631	0.000601361	0.104312988
[13,15]	0.000000419	0.145889042	0.000569711	0.096302805
[13,16]	0.000000069	0.138025658	0.000538060	0.081536302
[13,17]	0.000000011	0.129862861	0.000506409	0.063839670
[13,18]	0.000000003	0.121415528	0.000474759	0.046209302
[13,19]	0.000000002	0.112486355	0.000443108	0.030882462
[13,20]	0.000000003	0.102699196	0.000411458	0.019121057
[14,14]	0.000001556	0.167552459	0.000647620	0.159653367
[14,15]	0.000000321	0.159691066	0.000613535	0.155714172
[14,16]	0.000000055	0.151115518	0.000579449	0.134321498
[14,17]	0.000000009	0.142190642	0.000545364	0.105395412
[14,18]	0.000000002	0.132948746	0.000511279	0.075927853
[14,19]	0.000000001	0.123183887	0.000477194	0.050356919
[15,15]	0.000000224	0.172286693	0.000657358	0.217937615
[15,16]	0.000000042	0.163067420	0.000620839	0.198429781
[15,17]	0.000000007	0.153448263	0.000584319	0.158821745
[15,18]	0.000000002	0.143480800	0.000547799	0.114870274
[16,16]	0.000000033	0.174374295	0.000662228	0.254898211
[16,17]	0.000000010	0.164101126	0.000623273	0.215205017

[R, S]	DISTRIBUTION 9	DISTRIBUTION 10	DISTRIBUTION 11
[1, 1]	1.817768299	0.483425471	0.195293838
[1, 2]	0.484054844	0.128732261	0.052024353
[1, 3]	0.095974956	0.025529071	0.010429291
[1, 4]	0.017488490	0.004681963	0.002353522
[1, 5]	0.005368867	0.001566708	0.001990706
[1, 6]	0.003868071	0.001512380	0.003736282
[1, 7]	0.003939300	0.002409214	0.006834078
[1, 8]	0.004825978	0.004466649	0.011006454
[1, 9]	0.007230670	0.008227433	0.015661904
[1,10]	0.012384762	0.014035788	0.020006105
[1,11]	0.021633330	0.021705722	0.023363850
[1,12]	0.035692989	0.030396621	0.025413063
[1,13]	0.053737417	0.038800728	0.026206619
[1,14]	0.072892182	0.045560684	0.026029808
[1,15]	0.088732345	0.049678192	0.025221363
[1,16]	0.096847379	0.050706144	0.024052983
[1,17]	0.094761822	0.048699708	0.022686836
[1,18]	0.083100981	0.044063614	0.021182304
[1,19]	0.065276975	0.037441192	0.019519823
[1,20]	0.045903280	0.029670478	0.017630150
[1,21]	0.028911074	0.021721616	0.015435489
[1,22]	0.016379843	0.014536984	0.012907922
[1,23]	0.008475648	0.008805026	0.010129567
[1,24]	0.004172914	0.004793620	0.007315354
[1,25]	0.002128839	0.002350795	0.004762899
[1,26]	0.001253998	0.001066364	0.002738579
[1,27]	0.000890176	0.000486719	0.001365593
[1,28]	0.000784796	0.000277614	0.000594222
[1,29]	0.001243056	0.000345328	0.000294947
[1,30]	0.004061152	0.001082342	0.000474054
[1,31]	0.013471042	0.003582793	0.001453257
[1,32]	0.026882635	0.007149295	0.002888937
[2, 2]	0.494212417	0.131433656	0.053118789
[2, 3]	0.103774777	0.027600343	0.011205584
[2, 4]	0.017726173	0.004724766	0.002089988
[2, 5]	0.003843933	0.001070390	0.000958712
[2, 6]	0.002070980	0.000725974	0.001550595
[2, 7]	0.001953655	0.001037634	0.002894889
[2, 8]	0.002267266	0.001873798	0.004841771
[2, 9]	0.003219861	0.003490386	0.007151868
[2,10]	0.005375914	0.006112607	0.009441206
[2,11]	0.009429088	0.009739907	0.011328452
[2,12]	0.015871842	0.014040916	0.012582112
[2,13]	0.024528308	0.018399520	0.013170458
[2,14]	0.034205533	0.022103923	0.013210491
[2,15]	0.042796187	0.024572171	0.012875010
[2,16]	0.047962217	0.025491068	0.012317525
[2,17]	0.048129093	0.024824452	0.011638012
[2,18]	0.043229871	0.022743253	0.010880249
[2,19]	0.034734200	0.019558685	0.010042885
[2,20]	0.024946448	0.015690120	0.009094732
[2,21]	0.016015857	0.011634219	0.007995593
[2,22]	0.009220723	0.007889867	0.006725885
[2,23]	0.004819896	0.004842751	0.005318976
[2,24]	0.002369636	0.002669114	0.003877144
[2,25]	0.001184976	0.001320862	0.002551015
[2,26]	0.000674130	0.000599679	0.001483179
[2,27]	0.000463990	0.000269302	0.000747430
[2,28]	0.000399530	0.000147158	0.000327011
[2,29]	0.000614074	0.000172222	0.000157434
[2,30]	0.001989081	0.000530448	0.000236406
[2,31]	0.006664265	0.001772617	0.000722648
[3, 3]	0.078363558	0.020844647	0.008538622
[3, 4]	0.013755723	0.003666231	0.001666953
[3, 5]	0.002285790	0.000627346	0.000565840
[3, 6]	0.000751493	0.000253967	0.000640450
[3, 7]	0.000603125	0.000305217	0.001018212
[3, 8]	0.000672359	0.000523169	0.001581232

[R, S]	DISTRIBUTION 9 (CONT.)	DISTRIBUTION 10 (CONT.)	DISTRIBUTION 11 (CONT.)
[3, 9]	0.000919970	0.000956911	0.002266812
[3,10]	0.001492933	0.001677950	0.002971622
[3,11]	0.002592723	0.002704499	0.003579537
[3,12]	0.004383892	0.003961800	0.004007371
[3,13]	0.006860832	0.005283075	0.004229455
[3,14]	0.009730130	0.006455961	0.004270752
[3,15]	0.012407451	0.007290031	0.004181198
[3,16]	0.014187258	0.007667784	0.004010334
[3,17]	0.014532008	0.007558036	0.003793031
[3,18]	0.013324461	0.007000215	0.003546222
[3,19]	0.010926331	0.006083132	0.003271665
[3,20]	0.008004753	0.004932128	0.002960985
[3,21]	0.005236826	0.003698639	0.002602537
[3,22]	0.003065971	0.002538566	0.002190691
[3,23]	0.001622569	0.001577701	0.001735880
[3,24]	0.000800021	0.000880180	0.001269849
[3,25]	0.000394552	0.000439939	0.000839863
[3,26]	0.000217717	0.000200452	0.000491553
[3,27]	0.000145346	0.000089040	0.000249598
[3,28]	0.000122378	0.000046969	0.000110002
[3,29]	0.000183447	0.000052119	0.000052959
[3,30]	0.000589398	0.000157771	0.000078062
[4, 4]	0.009480394	0.002549629	0.001577164
[4, 5]	0.001503907	0.000444030	0.000954821
[4, 6]	0.000326947	0.000156870	0.001089495
[4, 7]	0.000205014	0.000171379	0.001360862
[4, 8]	0.000231605	0.000261680	0.001658033
[4, 9]	0.000329942	0.000421339	0.001945638
[4,10]	0.000538777	0.000663922	0.002192661
[4,11]	0.000912918	0.000987676	0.002372995
[4,12]	0.001488736	0.001365764	0.002473415
[4,13]	0.002247731	0.001748658	0.002496336
[4,14]	0.003090071	0.002077247	0.002455840
[4,15]	0.003841981	0.002300102	0.002370189
[4,16]	0.004307833	0.002386145	0.002255031
[4,17]	0.004349005	0.002328067	0.002119667
[4,18]	0.003947766	0.002138638	0.001966486
[4,19]	0.003217238	0.001845448	0.001792642
[4,20]	0.002350222	0.001487130	0.001593237
[4,21]	0.001537616	0.001109521	0.001365574
[4,22]	0.000902621	0.000758622	0.001113613
[4,23]	0.000480112	0.000470456	0.000850732
[4,24]	0.000238416	0.000262426	0.000598535
[4,25]	0.000118523	0.000131494	0.000380866
[4,26]	0.000065803	0.000060282	0.000215229
[4,27]	0.000044062	0.000027091	0.000106457
[4,28]	0.000037328	0.000014646	0.000047180
[4,29]	0.000056897	0.000017043	0.000027310
[5, 5]	0.001044262	0.000429107	0.002210610
[5, 6]	0.000248029	0.000287644	0.002863786
[5, 7]	0.000166060	0.000354373	0.003546333
[5, 8]	0.000199702	0.000479439	0.004146738
[5, 9]	0.000292736	0.000652239	0.004612695
[5,10]	0.000471661	0.000872196	0.004916679
[5,11]	0.000767635	0.001125212	0.005057536
[5,12]	0.001188581	0.001381873	0.005057331
[5,13]	0.001697469	0.001604596	0.004950723
[5,14]	0.002206044	0.001758738	0.004772862
[5,15]	0.002594961	0.001821833	0.004550871
[5,16]	0.002756951	0.001787062	0.004300171
[5,17]	0.002642657	0.001661363	0.004024337
[5,18]	0.002283257	0.001461527	0.003717023
[5,19]	0.001776265	0.001211174	0.003365853
[5,20]	0.001243089	0.000938686	0.002958968
[5,21]	0.000782833	0.000674192	0.002493974
[5,22]	0.000445548	0.000444345	0.001986243
[5,23]	0.000232702	0.000266335	0.001471268
[5,24]	0.000116135	0.000144428	0.000997048

[R, S]	DISTRIBUTION 9 (CONT.)	DISTRIBUTION 10 (CONT.)	DISTRIBUTION 11 (CONT.)
[5,25]	0.000060142	0.000071251	0.000608013
[5,26]	0.000035806	0.000033050	0.000328570
[5,27]	0.000025555	0.000015844	0.000156152
[5,28]	0.000022836	0.000010021	0.000068813
[6, 6]	0.000306984	0.000726243	0.006319782
[6, 7]	0.000293779	0.000985572	0.008143994
[6, 8]	0.000365573	0.001294535	0.009677564
[6, 9]	0.000495008	0.001644701	0.010819591
[6,10]	0.000713278	0.002026261	0.011526830
[6,11]	0.001053094	0.002411164	0.011822042
[6,12]	0.001518961	0.002754609	0.011779502
[6,13]	0.002063688	0.003006498	0.011494999
[6,14]	0.002584701	0.003126351	0.011056861
[6,15]	0.002951127	0.003094235	0.010528656
[6,16]	0.003054419	0.002913566	0.009944075
[6,17]	0.002856931	0.002607097	0.009308970
[6,18]	0.002410042	0.002210491	0.008606426
[6,19]	0.001830754	0.001766742	0.007805165
[6,20]	0.001251198	0.001321407	0.006874142
[6,21]	0.000770093	0.000916806	0.005803837
[6,22]	0.000429569	0.000584756	0.004627278
[6,23]	0.000221547	0.000340364	0.003427246
[6,24]	0.000111010	0.000180507	0.002318593
[6,25]	0.000059227	0.000088423	0.001408798
[6,26]	0.000036964	0.000042016	0.000757091
[6,27]	0.000027479	0.000021724	0.000357226
[7, 7]	0.000683293	0.002432769	0.015461473
[7, 8]	0.000903982	0.003300267	0.018998881
[7, 9]	0.001165217	0.004182492	0.021621271
[7,10]	0.001497375	0.005046152	0.023254045
[7,11]	0.001934084	0.005831646	0.023967227
[7,12]	0.002480200	0.006459999	0.023938535
[7,13]	0.003082805	0.006851358	0.023385801
[7,14]	0.003625995	0.006946638	0.022505208
[7,15]	0.003963284	0.006723252	0.021436638
[7,16]	0.003977785	0.006200200	0.020255441
[7,17]	0.003638328	0.005433701	0.018978429
[7,18]	0.003017874	0.004507302	0.017574460
[7,19]	0.002262595	0.003519408	0.015979942
[7,20]	0.001530526	0.002568883	0.014126469
[7,21]	0.000934978	0.001739167	0.011983985
[7,22]	0.000519637	0.001083740	0.009607222
[7,23]	0.000268832	0.000618244	0.007156858
[7,24]	0.000136721	0.000323472	0.004868608
[7,25]	0.000075083	0.000158394	0.002972405
[7,26]	0.000048428	0.000077036	0.001602916
[8, 8]	0.002235916	0.007195237	0.031687570
[8, 9]	0.002979796	0.009452158	0.037037681
[8,10]	0.003710298	0.011552206	0.040427818
[8,11]	0.004434550	0.013363262	0.042012349
[8,12]	0.005144789	0.014733443	0.042150489
[8,13]	0.005784934	0.015523432	0.041273316
[8,14]	0.006238699	0.015638448	0.039765490
[8,15]	0.006362997	0.015050084	0.037901357
[8,16]	0.006056482	0.013805843	0.035832450
[8,17]	0.005322441	0.012028360	0.033601467
[8,18]	0.004284186	0.009904360	0.031162571
[8,19]	0.003141060	0.007660722	0.028406979
[8,20]	0.002091261	0.005527172	0.025207855
[8,21]	0.001265472	0.003692579	0.021494639
[8,22]	0.000702436	0.002268879	0.017339804
[8,23]	0.000367612	0.001277090	0.013007961
[8,24]	0.000192759	0.000661221	0.008913288
[8,25]	0.000111085	0.000322655	0.005479575
[9, 9]	0.006882695	0.018330912	0.055394087
[9,10]	0.008938562	0.023209030	0.061679631
[9,11]	0.010657534	0.027337921	0.064806885
[9,12]	0.011940632	0.030423556	0.065413098

[R, S]	DISTRIBUTION 9 (CONT.)	DISTRIBUTION 10 (CONT.)	DISTRIBUTION 11 (CONT.)
[9,13]	0.012701313	0.032219847	0.064255321
[9,14]	0.012849173	0.032575003	0.062006369
[9,15]	0.012307968	0.031455249	0.059147225
[9,16]	0.011075662	0.028955961	0.055949519
[9,17]	0.009281576	0.025310737	0.052503474
[9,18]	0.007185944	0.020890532	0.048754106
[9,19]	0.005108236	0.016170538	0.044540461
[9,20]	0.003322875	0.011652367	0.039660272
[9,21]	0.001981227	0.007758741	0.033981294
[9,22]	0.001095946	0.004742701	0.027580757
[9,23]	0.000581561	0.002652232	0.020838000
[9,24]	0.000316593	0.001363572	0.014388239
[10,10]	0.018762487	0.040485138	0.084233997
[10,11]	0.023520203	0.049217885	0.089741369
[10,12]	0.026804583	0.055783569	0.091263070
[10,13]	0.028329006	0.059764409	0.090000937
[10,14]	0.028016754	0.060936487	0.087021057
[10,15]	0.025993973	0.059280263	0.083087420
[10,16]	0.022580788	0.054974776	0.078639018
[10,17]	0.018280006	0.048416808	0.073838991
[10,18]	0.013712907	0.040252491	0.068634503
[10,19]	0.009484859	0.031356530	0.062814064
[10,20]	0.006033694	0.022705073	0.056093004
[10,21]	0.003540825	0.015162437	0.048262770
[10,22]	0.001945861	0.009275613	0.039388546
[10,23]	0.001040664	0.005179437	0.029956952
[11,11]	0.044747878	0.078034655	0.113799640
[11,12]	0.053799617	0.090831737	0.116772497
[11,13]	0.058412305	0.098924539	0.115694833
[11,14]	0.058269976	0.102020894	0.112121725
[11,15]	0.053837190	0.100177158	0.107169844
[11,16]	0.046197770	0.093731636	0.101489111
[11,17]	0.036789575	0.083311326	0.095340320
[11,18]	0.027112209	0.069917999	0.088686909
[11,19]	0.018431338	0.054966411	0.081275611
[11,20]	0.011542853	0.040129074	0.072743451
[11,21]	0.006686919	0.026976994	0.062802277
[11,22]	0.003643647	0.016578727	0.051492351
[12,12]	0.093033453	0.132171479	0.139687287
[12,13]	0.106569963	0.147091736	0.139133868
[12,14]	0.109656213	0.153836296	0.135187468
[12,15]	0.102907762	0.152654743	0.129372743
[12,16]	0.088728614	0.144184707	0.122585336
[12,17]	0.070494934	0.129388369	0.115204106
[12,18]	0.051616536	0.109690799	0.107222168
[12,19]	0.034791576	0.087133729	0.098356191
[12,20]	0.021585862	0.064254641	0.088176117
[12,21]	0.012386573	0.043584066	0.076321126
[13,13]	0.168104116	0.198257264	0.159323142
[13,14]	0.182206015	0.210933592	0.155240583
[13,15]	0.176566152	0.211795552	0.148755293
[13,16]	0.155072394	0.201986326	0.141032903
[13,17]	0.124288662	0.182967533	0.132584851
[13,18]	0.091201885	0.156665967	0.123444258
[13,19]	0.061347514	0.125776121	0.113308404
[13,20]	0.037880349	0.093758514	0.101692878
[14,14]	0.263078487	0.265681326	0.172366901
[14,15]	0.268009559	0.270416932	0.165386231
[14,16]	0.243031556	0.260505911	0.156891464
[14,17]	0.198612341	0.238123782	0.147535596
[14,18]	0.147256073	0.205812981	0.137398455
[14,19]	0.099435372	0.166919562	0.126165557
[15,15]	0.355388655	0.320899067	0.179886326
[15,16]	0.338234219	0.312598361	0.170744788
[15,17]	0.285295168	0.288333052	0.160604803
[15,18]	0.215739352	0.251401940	0.149596615
[16,16]	0.413345032	0.351926717	0.183226563
[16,17]	0.365590659	0.327831432	0.172392824

APPENDIX 10

THE COMPUTER PROGRAM FOR CALCULATING THE BEST LINEAR ESTIMATORS FOR THE NEAR-NORMAL DISTRIBUTIONS

This appendix gives the Algol program used to calculate the best linear estimator for sample size 8. The program was run on the Burroughs B6718 computer at the University of Canterbury.

SYMLINEQNIMPRV, which was used to solve symmetric simultaneous linear equations, is from the Burroughs Numerals Numerical Analysis Program Library. After solving the equations it improves the solutions iteratively and outputs the number of digits correct in the initial solution. This number can be taken as a measure of how well-conditioned the equations are, and was always satisfactorily high (around 10) when the results in Appendices 2 and 7 were calculated.

Other checks were made on the accuracy of the results. The sum of the elements of the covariance matrix should equal the sample size times the variance of the distribution,and the coefficients of the best linear location estimator should sum to 1. The sum of the covariances was usually correct to 8 or more decimal places,and the sum of the coefficients to 9 or more decimal places.

Note that the names some variables have in the program are different from those they have in Appendix 1. For example the slope of a line segment of a density is called A in Appendix 1 and DENSA in the program.

The parameters of the distributions were initially set with the aid of a hand calculator. In the computer program these parameters are adjusted slightly so that the probability mass and the variance of the distributions are 1 to the accuracy of the computer rather than merely to the accuracy of the calculator. For some distributions the method used to make this adjustment was different from the one in the program given here. The values of the parameters given in Appendix 1 are the adjusted ones.

```
BEGIN
   INTEGER N,REGS,D,NUMDISTS;
   FILE CARD(KIND=READER);
   N:=8;
   READ(CARD,<I2>,NUMDISTS);
   FOR D:=1 STEP 1 UNTIL NUMDISTS DO
   BEGIN
      READ(CARD,<I2>,REGS);
      BEGIN
         FILE LP(KIND=PRINTER);
         FILE RESULTS8(KIND=DISK,FILETYPE=5,MINRECSIZE=1,MAXRECSIZE=30,
         PROTECTION=SAVE,BLOCKSIZE=300,AREASIZE=1,AREAS=100,INTMODE=SINGLE);
         FILE COUNTR8(KIND=DISK,FILETYPE=0,MAXRECSIZE=30,PROTECTION=SAVE,
         BLOCKSIZE=30,AREASIZE=30,AREAS=1);
         INTEGER R,S,IX,IY,K;
         INTEGER COMBSM,COMBSC;
         INTEGER RERUN;
         REAL X,Y,INTI,INTT,INTR;
         REAL PROBDIST,VARDIST,SUMCOVS,SUMCOEFFS,STDEVDIST,KURTOSIS;
         REAL CUT1,CUT2,REGCUT1,REGCUT2,DENSCUT1,DENSCUT2;
         REAL PI,NORMPROB1,NORMPROB2;
         REAL PARTPROB,PARTVAR;
         REAL CENTRALPROB,CENTRALVAR,MEDIUMPROB,MEDIUMVAR,EXTREMEPROB,EXTREMEVAR;
         REAL CENTRALNORMALPROB,MEDIUMNORMALPROB,EXTREMENORMALPROB;
         REAL CENTRALNORMALVAR,MEDIUMNORMALVAR,EXTREMENORMALVAR;
         REAL CENTRALPROBDIFF,MEDIUMPROBDIFF,EXTREMEPROBDIFF;
         REAL CENTRALVARDIFF,MEDIUMVARDIFF,EXTREMEVARDIFF;
         REAL NORMALIZER,NORMALIZERR;
         REAL DIGITS,DIGITSR;
         INTEGER ARRAY COUNT[1:1];
         REAL ARRAY DENSA,DENSB[1:REGS],CUM,JUMP[1:REGS+1];
         REAL ARRAY LHGT,RHGT[1:REGS];
         REAL ARRAY PROBREG,VARREG[1:REGS];
         REAL ARRAY CUT,NORMPROB[1:2];
         REAL ARRAY MEAN[0:N],COVARIANCE[0:N,0:N];
         REAL ARRAY AONES,ONEVINV,MEANVINV[0:N];
         REAL ARRAY COEFF,COEFFR[1:N];
         REAL ARRAY RUBBISH[1:1];
         REAL ARRAY CONREGS[1:3];
         REAL ARRAY NORM[1:2],COEFFS[1:2*((N+1) DIV 2)];
         REAL ARRAY PARTCOV1[1:8],PARTCOV2[2:7],PARTCOV3[3:6],PARTCOV4[4:5];
         LABEL SINGMATRIX;


         COMMENT THE PDF IS DENSA[IX]*(X-JUMP[IX])+DENSB[IX] FROM JUMP[IX] TO
         JUMP[IX+1].CUM[I] IS THE CUMALATIVE PROBABILITY AT JUMP[I].REGS IS THE
         NUMBER OF REGIONS IN THE DEFINITION OF THE PDF;
```

```
REAL PROCEDURE FM(X,N,R,IX,REGS,DENSA,DENSB,CUM,JUMP);
COMMENT DEFINES THE FUNCTION TO BE INTEGRATED IN CALCULATING THE
MEANS OF THE ORDER STATISTICS;
VALUE X,N,R,IX,REGS;
INTEGER N,R,IX,REGS;
REAL X; REAL ARRAY DENSA,DENSB,CUM,JUMP[*];
BEGIN
   FM:=X*(DENSA[IX]*X+DENSB[IX])*(1/2*DENSA[IX]*
   (X**2-JUMP[IX]**2)+DENSB[IX]*(X-JUMP[IX])+CUM[IX])**(R-1)*
   (1-1/2*DENSA[IX]*(X**2-JUMP[IX]**2)-DENSB[IX]*(X-JUMP[IX])-CUM[IX])**
   (N-R);
END;


REAL PROCEDURE FV(X,N,R,IX,REGS,DENSA,DENSB,CUM,JUMP);
COMMENT DEFINES THE FUNCTION TO BE INTEGRATED IN CALCULATING THE
VARIANCES OF THE ORDER STATISTICS;
VALUE X,N,R,IX,REGS;
INTEGER N,R,IX,REGS;
REAL X; REAL ARRAY DENSA,DENSB,CUM,JUMP[*];
BEGIN
   FV:=(DENSA[IX]*X+DENSB[IX])*(1/2*DENSA[IX]*
   (X**2-JUMP[IX]**2)+DENSB[IX]*(X-JUMP[IX])+CUM[IX])**(R-1)*
   (1-1/2*DENSA[IX]*(X**2-JUMP[IX]**2)-DENSB[IX]*(X-JUMP[IX])-CUM[IX])**
   (N-R)*X**2;
END;


REAL PROCEDURE FC(X,Y,N,R,S,IX,IY,REGS,DENSA,DENSB,CUM,JUMP);
COMMENT DEFINES THE FUNCTION TO BE INTEGRATED IN CALCULATING THE
COVARIANCES OF THE ORDER STATISTICS;
VALUE X,Y,N,R,S,IX,IY,REGS;
INTEGER N,R,S,IX,IY,REGS;
REAL X,Y; REAL ARRAY DENSA,DENSB,CUM,JUMP[*];
BEGIN
   FC:=X*Y*(DENSA[IX]*X+DENSB[IX])*(DENSA[IY]*Y+
   DENSB[IY])*(1/2*DENSA[IX]*(X**2-JUMP[IX]**2)+DENSB[IX]*(X-JUMP[IX])
   +CUM[IX])**(R-1)*(1/2*DENSA[IY]*(Y**2-JUMP[IY]**2)+DENSB[IY]*
   (Y-JUMP[IY])+CUM[IY]-1/2*DENSA[IX]*(X**2-JUMP[IX]**2)-DENSB[IX]
   *(X-JUMP[IX])-CUM[IX])**(S-R-1)*(1-1/2*DENSA[IY]*(Y**2-JUMP[IY]**2)-
   DENSB[IY]*(Y-JUMP[IY])-CUM[IY])**(N-S);
END;


PROCEDURE COMBSMEAN(N,R);
COMMENT EVALUATES N*(N-1) C(R-1);
VALUE N,R; INTEGER N,R;
BEGIN
   INTEGER COUNTER,NUMER,DENOM,I,J;
   COUNTER:=INTEGER(MIN((N-R),(R-1)));
   NUMER:=N;
   FOR I:=1 STEP 1 UNTIL COUNTER DO NUMER:=NUMER*(N-I);
   DENOM:=1;
   FOR J:=1 STEP 1 UNTIL COUNTER DO DENOM:=DENOM*J;
   COMBSM:=INTEGER(NUMER/DENOM);
END;
```

```
PROCEDURE COMBSCOV(N,R,S);
COMMENT EVALUATES NFAC/((R-1) FAC(S-R-1) FAC(N-S) FAC);
VALUE N,R,S; INTEGER N,R,S;
BEGIN
   INTEGER A,B,C,NUMER,FACB,FACC,I,J;
   A:=R-1;
   B:=S-R-1;
   C:=N-S;
   IF A<B THEN
   BEGIN
      A:=B;
      B:=R-1
   END;
   IF A<C THEN
   BEGIN
      C:=A;
      A:=N-S
   END;
   NUMER:=1;
   FOR I:=1 STEP 1 UNTIL N-A DO NUMER:=NUMER*(N-I+1);
   FACB:=1;
   FOR I:=1 STEP 1 UNTIL B DO FACB:=FACB*I;
   FACC:=1;
   FOR J:=1 STEP 1 UNTIL C DO FACC:=FACC*J;
   COMBSC:=INTEGER((NUMER/FACB)/FACC)
END;


PROCEDURE INTINT(F,A,B);
COMMENT APPROXIMATES THE INTEGRAL OF F(X) OVER AN INTERVAL FROM A TO B.
A 19TH DEGREE 10 POINT GAUSSIAN FORMULA IS USED.;
VALUE A,B; REAL A,B;

REAL PROCEDURE F;
BEGIN
   REAL Q,X; INTEGER I; REAL ARRAY P,W[1:10];
   P[1]:=-P[10]:=0.97390652851; P[2]:=-P[9]:=0.86506336668;
   P[3]:=-P[8]:=0.67940956829; P[4]:=-P[7]:=0.43339539412;
   P[5]:=-P[6]:=0.14887433898;
   W[1]:=W[10]:=0.066671344308; W[2]:=W[9]:=0.14945134915;
   W[3]:=W[8]:=0.21908636251; W[4]:=W[7]:=0.26926671930;
   W[5]:=W[6]:=0.29552422471;
   Q:=0;
   FOR I:=1 STEP 1 UNTIL 10 DO
   BEGIN
      X:=1/2*(A+B)+1/2*(B-A)*P[I];
      Q:=Q+W[I]*F(X,N,R,IX,REGS,DENSA,DENSB,CUM,JUMP)
   END;
   INTI:=1/2*(B-A)*Q;
END;
```

```
PROCEDURE INTREC(F,A,B,C,D);
COMMENT APPROXIMATES THE INTEGRAL OF F(X,Y) OVER A RECTANGLE FROM X=A TO
AND FROM Y=C TO Y=D.A 19TH DEGREE 100 POINT GAUSSIAN FORMULA IS USED;
VALUE A,B,C,D; REAL A,B,C,D;

REAL PROCEDURE F;
BEGIN
   REAL Q,X,Y; INTEGER I,J; REAL ARRAY P,W[1:10];
   P[1]:=-P[10]:=0.97390652851; P[2]:=-P[9]:=0.86506336668;
   P[3]:=-P[8]:=0.67940956829; P[4]:=-P[7]:=0.43339539412;
   P[5]:=-P[6]:=0.14887433898;
   W[1]:=W[10]:=0.066671344308; W[2]:=W[9]:=0.14945134915;
   W[3]:=W[8]:=0.21908636251; W[4]:=W[7]:=0.26926671930;
   W[5]:=W[6]:=0.29552422471;
   Q:=0;
   FOR I:=1 STEP 1 UNTIL 10 DO
   BEGIN
      Y:=1/2*(C+D)+1/2*(D-C)*P[I];
      FOR J:=1 STEP 1 UNTIL 10 DO
      BEGIN
         X:=1/2*(A+B)+1/2*(B-A)*P[J];
         Q:=Q+W[I]*W[J]*F(X,Y,N,R,S,IX,IY,REGS,DENSA,DENSB,CUM,JUMP)
      END;
   END;
   INTR:=1/4*(B-A)*(D-C)*Q;
END;


PROCEDURE INTTRI(F,U1,U2,V1,V2,W1,W2);
COMMENT APPROXIMATES THE INTEGRAL OVER A TRIANGLE WITH VERTICES (U1,U2),
((V1,V2),(W1,W2).AN 18TH DEGREE 100 POINT GAUSSIAN FORMULA IS USED;
VALUE U1,U2,V1,V2,W1,W2; REAL U1,U2,V1,V2,W1,W2;

REAL PROCEDURE F;
BEGIN
   REAL Q,X1,X2; INTEGER I,J;
   REAL ARRAY P,W[1:10];
   P[1]:=0.013046735741;P[2]:=0.067468316656;P[3]:=0.16029521585;
   P[4]:=0.28330230294;P[5]:=0.42556283051;P[6]:=0.57443716949;
   P[7]:=0.71669769706;P[8]:=0.83970478415;P[9]:=0.93253168334;
   P[10]:=0.98695326426;
   W[1]:=W[10]:=0.033335672154; W[2]:=W[9]:=0.074725674575;
   W[3]:=W[8]:=0.10954318126; W[4]:=W[7]:=0.13463335965;
   W[5]:=W[6]:=0.14776211236;
   Q:=0;
   FOR I:=1 STEP 1 UNTIL 10 DO
   FOR J:=1 STEP 1 UNTIL 10 DO
   BEGIN
      X1:=(V1-U1)*P[I]+(W1-U1)*P[J]*(1-P[I])+U1;
      X2:=(V2-U2)*P[I]+(W2-U2)*P[J]*(1-P[I])+U2;
      Q:=Q+W[I]*W[J]*F(X1,X2,N,R,S,IX,IY,REGS,DENSA,DENSB,CUM,JUMP)*(1-P[I])
   END;
   INTT:=ABS((V1-U1)*(W2-U2)-(W1-U1)*(V2-U2))*Q
END;
```

```
COMMENT THE DISTRIBUTION IS READ IN AND DETAILS ARE CALCULATED AND
WRITTEN OUT;
FOR IX:=1 STEP 1 UNTIL(REGS+2) DIV 2 DO
READ(CARD,/,JUMP[IX],DENSA[IX],DENSB[IX]);
READ(CARD,/,CUT1,CUT2);
READ(CARD,/,NORMPROB1,NORMPROB2);
FOR IX:=(REGS+4) DIV 2 STEP 1 UNTIL REGS+1 DO JUMP[IX]:=
-JUMP[REGS+2-IX];
FOR IX:=(REGS+3) DIV 2 STEP 1 UNTIL REGS DO DENSA[IX]:=
-DENSA[REGS+1-IX];
FOR IX:=(REGS+3) DIV 2 STEP 1 UNTIL REGS DO DENSB[IX]:=
DENSB[REGS+1-IX];
WRITE(LP,<"THE PARAMETERS OF THE DISTRIBUTION ARE:">);
FOR IX:=1 STEP 1 UNTIL REGS DO
WRITE(LP,*//[30],JUMP[IX],DENSA[IX],DENSB[IX]);
WRITE(LP[SPACE 2],*//[30],JUMP[REGS+1]);
FOR IX:=(REGS+2) DIV 2,REGS DO
BEGIN
   PROBREG[IX]:=DENSB[IX]*(JUMP[IX+1]-JUMP[IX])+DENSA[IX]*
   (JUMP[IX+1]**2-JUMP[IX]**2)/2;
   VARREG[IX]:=DENSA[IX]/4*(JUMP[IX+1]**4-JUMP[IX]**4)
   +DENSB[IX]/3*(JUMP[IX+1]**3-JUMP[IX]**3);
   LHGT[IX]:=DENSA[IX]*JUMP[IX]+DENSB[IX];
   RHGT[IX]:=DENSA[IX]*JUMP[IX+1]+DENSB[IX];
   WRITE(LP,*//[30],LHGT[IX],RHGT[IX],PROBREG[IX],VARREG[IX]);
END;
SPACE(LP,2);
CUM[1]:=0;
FOR IX:=2 STEP 1 UNTIL REGS+1 DO
CUM[IX]:=CUM[IX-1]+DENSB[IX-1]*(JUMP[IX]-JUMP[IX-1])+DENSA[IX-1]*
(JUMP[IX]**2-JUMP[IX-1]**2)/2;
PROBDIST:=CUM[REGS+1];
WRITE(LP,*//,PROBDIST);
VARDIST:=0;
FOR IX:=1 STEP 1 UNTIL REGS DO VARDIST:=VARDIST+DENSA[IX]/4*
(JUMP[IX+1]**4-JUMP[IX]**4)+DENSB[IX]/3*(JUMP[IX+1]**3-JUMP[IX]**3);
WRITE(LP[SPACE 4],*//,VARDIST);


COMMENT THE DISTRIBUTION IS ADJUSTED SO THAT THE PROBABILITY AND
VARIANCE ARE ONE TO MACHINE ACCURRACY;
PARTPROB:=0;
FOR IX:=(REGS+6) DIV 2 STEP 1 UNTIL REGS+1 DO
PARTPROB:=PARTPROB+DENSB[IX-1]*(JUMP[IX]-JUMP[IX-1])+DENSA[IX-1]*
(JUMP[IX]**2-JUMP[IX-1]**2)/2;
PARTPROB:=(1/2)-PARTPROB;
IX:=(REGS+2) DIV 2;
LHGT[IX]:=(2*PARTPROB)/(JUMP[IX+1]-JUMP[IX])-RHGT[IX];
DENSA[IX]:=(RHGT[IX]-LHGT[IX])/(JUMP[IX+1]-JUMP[IX]);
DENSA[IX-1]:=-DENSA[IX];
DENSB[IX]:=DENSB[IX-1]:=LHGT[IX];
RHGT[IX-1]:=DENSB[IX-1];
WRITE(LP,<"AFTER ADJUSTMENT OF THE PROBABILITY THE PARAMETERS ARE:">);
WRITE(LP,*//[30],LHGT[IX],RHGT[IX],DENSA[IX],DENSB[IX]);
CUM[1]:=0;
FOR IX:=2 STEP 1 UNTIL REGS+1 DO
CUM[IX]:=CUM[IX-1]+DENSB[IX-1]*(JUMP[IX]-JUMP[IX-1])+DENSA[IX-1]*
(JUMP[IX]**2-JUMP[IX-1]**2)/2;
PROBDIST:=CUM[REGS+1];
WRITE(LP,*//,PROBDIST);
PARTVAR:=0;
FOR IX:=((REGS+2) DIV 2) STEP 1 UNTIL REGS-1 DO PARTVAR:=PARTVAR
+DENSA[IX]/4*(JUMP[IX+1]**4-JUMP[IX]**4)+DENSB[IX]/3*
(JUMP[IX+1]**3-JUMP[IX]**3);
PARTVAR:=(1/2)-PARTVAR;
JUMP[1]:=-JUMP[REGS+1]:=SQRT(6*PARTVAR/PROBREG[REGS]-2*JUMP[REGS]**2)
-JUMP[REGS];
RHGT[1]:=LHGT[REGS]:=2*PROBREG[REGS]/(JUMP[REGS+1]-JUMP[REGS]);
DENSA[1]:=-DENSA[REGS]:=-LHGT[REGS]/(JUMP[REGS+1]-JUMP[REGS]);
DENSB[1]:=DENSB[REGS]:=LHGT[REGS]*JUMP[REGS+1]
/(JUMP[REGS+1]-JUMP[REGS]);
```

```
COMMENT THE DETAILS OF THE ADJUSTED DISTRIBUTION ARE CALCULATED AND
WRITTEN OUT;
WRITE(LP,<"AFTER ADJUSTMENT OF THE DISTRIBUTION,THE PARAMETERS ARE:">);
FOR IX:=1 STEP 1 UNTIL REGS DO
WRITE(LP,*//[30],JUMP[IX],DENSA[IX],DENSB[IX]);
WRITE(LP[SPACE 2],*//[30],JUMP[REGS+1]);
FOR IX:=1 STEP 1 UNTIL REGS DO
BEGIN
   LHGT[IX]:=DENSA[IX]*JUMP[IX]+DENSB[IX];
   RHGT[IX]:=DENSA[IX]*JUMP[IX+1]+DENSB[IX];
   PROBREG[IX]:=DENSB[IX]*(JUMP[IX+1]-JUMP[IX])+DENSA[IX]*
   (JUMP[IX+1]**2-JUMP[IX]**2)/2;
   VARREG[IX]:=DENSA[IX]/4*(JUMP[IX+1]**4-JUMP[IX]**4)
   +DENSB[IX]/3*(JUMP[IX+1]**3-JUMP[IX]**3);
   WRITE(LP,*//[30],LHGT[IX],RHGT[IX],PROBREG[IX],VARREG[IX]);
END;
SPACE(LP,2);
FOR IX:=(REGS+2) DIV 2 STEP 1 WHILE CUT1>JUMP[IX] DO REGCUT1:=IX;
FOR IX:=REGCUT1 STEP 1 WHILE CUT2>JUMP[IX] DO REGCUT2:=IX;
CENTRALPROB:=0; CENTRALVAR:=0;
FOR IX:=((REGS+2) DIV 2) STEP 1 UNTIL REGCUT1-1 DO
BEGIN
   CENTRALPROB:=CENTRALPROB+DENSB[IX]*(JUMP[IX+1]-JUMP[IX])
   +DENSA[IX]*(JUMP[IX+1]**2-JUMP[IX]**2)/2;
   CENTRALVAR:=CENTRALVAR + DENSA[IX]/4*(JUMP[IX+1]**4 -JUMP[IX]**4)
   +DENSB[IX]/3*(JUMP[IX+1]**3-JUMP[IX]**3);
END;
CENTRALPROB:=CENTRALPROB+DENSB[REGCUT1]*(CUT1-JUMP[REGCUT1])
+DENSA[REGCUT1]*(CUT1**2-JUMP[REGCUT1]**2)/2;
CENTRALVAR:=CENTRALVAR + DENSA[REGCUT1]/4*(CUT1**4 - JUMP[REGCUT1]**4)
+DENSB[REGCUT1]/3*(CUT1**3-JUMP[REGCUT1]**3);
IF REGCUT1 EQL REGCUT2 THEN
BEGIN
   MEDIUMPROB:=DENSB[REGCUT1]*(CUT2-CUT1)
   +DENSA[REGCUT1]/2*(CUT2**2-CUT1**2);
   MEDIUMVAR:=DENSA[REGCUT1]/4*(CUT2**4-CUT1**4)
   +DENSB[REGCUT1]/3*(CUT2**3-CUT1**3);
END
ELSE
BEGIN
   MEDIUMPROB:=DENSB[REGCUT1]*(JUMP[REGCUT1+1]-CUT1)
   +DENSA[REGCUT1]/2*(JUMP[REGCUT1+1]**2-CUT1**2);
   MEDIUMVAR:=DENSA[REGCUT1]/4*(JUMP[REGCUT1+1]**4-CUT1**4)
   +DENSB[REGCUT1]/3*(JUMP[REGCUT1+1]**3-CUT1**3);
   FOR IX:=REGCUT1+1 STEP 1 UNTIL REGCUT2-1 DO
   BEGIN
      MEDIUMPROB:=MEDIUMPROB+DENSB[IX]*(JUMP[IX+1]-JUMP[IX])
      +DENSA[IX]*(JUMP[IX+1]**2-JUMP[IX]**2)/2;
      MEDIUMVAR := MEDIUMVAR + DENSA[IX]/4*(JUMP[IX+1]**4 -JUMP[IX]**4)
      +DENSB[IX]/3*(JUMP[IX+1]**3-JUMP[IX]**3);
   END;
   MEDIUMPROB:=MEDIUMPROB+DENSB[REGCUT2]*(CUT2-JUMP[REGCUT2])
   +DENSA[REGCUT2]*(CUT2**2-JUMP[REGCUT2]**2)/2;
   MEDIUMVAR := MEDIUMVAR + DENSA[REGCUT2]/4*(CUT2**4 - JUMP[REGCUT2]**4)
   +DENSB[REGCUT2]/3*(CUT2**3-JUMP[REGCUT2]**3);
END;
EXTREMEPROB:=DENSB[REGCUT2]*(JUMP[REGCUT2+1]-CUT2)
+DENSA[REGCUT2]/2*(JUMP[REGCUT2+1]**2-CUT2**2);
EXTREMEVAR:=DENSA[REGCUT2]/4*(JUMP[REGCUT2+1]**4-CUT2**4)
+DENSB[REGCUT2]/3*(JUMP[REGCUT2+1]**3-CUT2**3);
FOR IX:=REGCUT2+1 STEP 1 UNTIL REGS DO
BEGIN
   EXTREMEPROB:=EXTREMEPROB+DENSB[IX]*(JUMP[IX+1]-JUMP[IX])
   +DENSA[IX]*(JUMP[IX+1]**2-JUMP[IX]**2)/2;
   EXTREMEVAR:=EXTREMEVAR + DENSA[IX]/4*(JUMP[IX+1]**4 -JUMP[IX]**4)
   +DENSB[IX]/3*(JUMP[IX+1]**3-JUMP[IX]**3);
END;
CENTRALNORMALPROB:=NORMPROB1/2;
MEDIUMNORMALPROB:=(NORMPROB2-NORMPROB1)/2;
EXTREMENORMALPROB:=(1-NORMPROB2)/2;
PI:=3.141592653590;
CENTRALNORMALVAR:=CENTRALNORMALPROB-(1/SQRT(2*PI))
*CUT1*EXP(-(CUT1**2)/2);
MEDIUMNORMALVAR:=MEDIUMNORMALPROB+(1/SQRT(2*PI))
*(CUT1*EXP(-(CUT1**2)/2)-CUT2*EXP(-(CUT2**2)/2));
EXTREMENORMALVAR:=EXTREMENORMALPROB+(1/SQRT(2*PI))
*CUT2*EXP(-(CUT2**2)/2);
```

```
CENTRALPROBDIFF:=CENTRALPROB-CENTRALNORMALPROB;
CENTRALVARDIFF:=CENTRALVAR-CENTRALNORMALVAR;
MEDIUMPROBDIFF:=MEDIUMPROB-MEDIUMNORMALPROB;
MEDIUMVARDIFF:=MEDIUMVAR-MEDIUMNORMALVAR;
EXTREMEPROBDIFF:=EXTREMEPROB-EXTREMENORMALPROB;
EXTREMEVARDIFF:=EXTREMEVAR-EXTREMENORMALVAR;
WRITE(LP,*//[40],CUT1,CUT2);
WRITE(LP,*//[40],NORMPROB1,NORMPROB2);
WRITE(LP,*//[40],CENTRALPROB,MEDIUMPROB,EXTREMEPROB);
WRITE(LP,*//[40],CENTRALNORMALPROB,MEDIUMNORMALPROB,EXTREMENORMALPROB);
WRITE(LP,*//[40],CENTRALPROBDIFF,MEDIUMPROBDIFF,EXTREMEPROBDIFF);
WRITE(LP,*//[40],CENTRALVAR,MEDIUMVAR,EXTREMEVAR);
WRITE(LP,*//[40],CENTRALNORMALVAR,MEDIUMNORMALVAR,EXTREMENORMALVAR);
WRITE(LP[SPACE 2],*//[40],CENTRALVARDIFF,MEDIUMVARDIFF,EXTREMEVARDIFF);
DENSCUT1:=DENSA[REGCUT1]*CUT1+DENSB[REGCUT1];
DENSCUT2:=DENSA[REGCUT2]*CUT2+DENSB[REGCUT2];
WRITE(LP[SPACE 2],*//[40],DENSCUT1,DENSCUT2);
FOR IX:=2 STEP 1 UNTIL REGS+1 DO
CUM[IX]:=CUM[IX-1]+DENSB[IX-1]*(JUMP[IX]-JUMP[IX-1])+DENSA[IX-1]*
(JUMP[IX]**2-JUMP[IX-1]**2)/2;
PROBDIST:=CUM[REGS+1];
WRITE(LP,*//,PROBDIST);
VARDIST:=0;
FOR IX:=1 STEP 1 UNTIL REGS DO VARDIST:=VARDIST+DENSA[IX]/4*
(JUMP[IX+1]**4-JUMP[IX]**4)+DENSB[IX]/3*(JUMP[IX+1]**3-JUMP[IX]**3);
WRITE(LP,*//,VARDIST);
KURTOSIS:=0;
FOR IX:=1 STEP 1 UNTIL REGS DO KURTOSIS:=KURTOSIS+DENSA[IX]/6*
(JUMP[IX+1]**6-JUMP[IX]**6)+DENSB[IX]/5*(JUMP[IX+1]**5-JUMP[IX]**5);
KURTOSIS:=KURTOSIS/VARDIST**2;
WRITE(LP[SPACE 4],*//,KURTOSIS);


COMMENT THE EXPECTATIONS AND COVARIANCES OF THE ORDER STATISTICS ARE
CALCULATED;
FOR R:=1 STEP 1 UNTIL(N+1) DIV 2 DO
BEGIN
   COMBSMEAN(N,R);
   MEAN[R]:=0;
   FOR IX:=1 STEP 1 UNTIL REGS DO
   BEGIN
      INTINT(FM,JUMP[IX],JUMP[IX+1]);
      MEAN[R]:=MEAN[R]+INTI
   END;
   MEAN[N+1-R]:=-MEAN[R]:=COMBSM*MEAN[R];
   COVARIANCE[R,R]:=0;
   FOR IX:=1 STEP 1 UNTIL REGS DO
   BEGIN
      INTINT(FV,JUMP[IX],JUMP[IX+1]);
      COVARIANCE[R,R]:=COVARIANCE[R,R]+INTI
   END;
   COVARIANCE[N+1-R,N+1-R]:=COVARIANCE[R,R]:=COMBSM*COVARIANCE[R,R]
   -MEAN[R]**2;
END;
FOR R:=1 STEP 1 UNTIL N DIV 2 DO
FOR S:=R+1 STEP 1 UNTIL N+1-R DO
BEGIN
   COMBSCOV(N,R,S);
   COVARIANCE[R,S]:=0;
   FOR IX:=1 STEP 1 UNTIL REGS DO
   BEGIN
      IY:=IX;
      INTTRI(FC,JUMP[IX],JUMP[IX],JUMP[IX],JUMP[IX+1],JUMP[IX+1],JUMP[IX+1]);
      COVARIANCE[R,S]:=COVARIANCE[R,S]+INTT;
      FOR IY:=IX+1 STEP 1 UNTIL REGS DO
      BEGIN
         INTREC(FC,JUMP[IX],JUMP[IX+1],JUMP[IY],JUMP[IY+1]);
         COVARIANCE[R,S]:=COVARIANCE[R,S]+INTR
      END;
   END;
   COVARIANCE[N+1-S,N+1-R]:=COVARIANCE[N+1-R,N+1-S]:=
   COVARIANCE[R,S]:=COVARIANCE[S,R]:=
   COMBSC*COVARIANCE[R,S]-MEAN[R]*MEAN[S];
   WRITE(LP,<2I3>,R,S);
   WRITE(LP,/,COVARIANCE[R,S]);
END;
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COMMENT THE COVARIANCE MATRIX AND RELATED DETAILS ARE WRITTEN OUT;
WRITE(LP,<"THE EXPECTATIONS OF THE ORDER STATISTICS ARE:">);
WRITE(LP[SPACE 2],<4(I2,X2,F13.9,X5)>,FOR R:=1 STEP 1 UNTIL N DO
[R,MEAN[R]]);
SPACE(LP,2);
WRITE(LP,<"THE COVARIANCE MATRIX IS:">);
FOR R:=1 STEP 1 UNTIL N DO
BEGIN
   WRITE(LP[SPACE 2]);
   WRITE(LP[SPACE 2],<4("(",I2,",",I2,")",F13.9,X5)>,FOR S:=1 STEP 1 UNTIL
   N DO[R,S,COVARIANCE[R,S]]);
END;
SPACE(LP,2);
WRITE(LP,<"THE CORRELATION MATRIX IS:">);
FOR R:=1 STEP 1 UNTIL N DO
BEGIN
   WRITE(LP[SPACE 2]);
   WRITE(LP[SPACE 2],<4("(",I2,",",I2,")",F13.9,X5)>,FOR S:=1 STEP 1 UNTIL
   N DO [R,S,COVARIANCE[R,S]/SQRT(COVARIANCE[R,R]*COVARIANCE[S,S])]);
END;
SPACE(LP,2);
SUMCOVS:=0;
FOR R:=1 STEP 1 UNTIL N DO
FOR S:=1 STEP 1 UNTIL N DO
SUMCOVS:=SUMCOVS+COVARIANCE[R,S];
WRITE(LP,<"THE SUM OF THE ELEMENTS OF THE COVARIANCE MATRIX IS
",F13.9>,SUMCOVS);
WRITE(LP[SPACE 2],<"THE PRODUCT OF THE SAMPLE SIZE AND THE VARIANCE OF T
HE DISTRIBUTION IS",F13.9>,N*VARDIST);


COMMENT THE LOCATION ESTIMATOR IS CALCULATED AND WRITTEN OUT;
FOR R:=1 STEP 1 UNTIL N DO AONES[R]:=1;
SYMLINEQNIMPRV(N,COVARIANCE,AONES,ONEVINV,DIGITS,SINGMATRIX);
NORMALIZER:=0;
FOR R:=1 STEP 1 UNTIL N DO NORMALIZER:=*+ONEVINV[R];
FOR R:=1 STEP 1 UNTIL N DO COEFF[R]:=ONEVINV[R]/NORMALIZER;
WRITE(LP[SPACE 3]);
WRITE(LP,<"THE COEFFICIENTS OF THE BLCOS ESTIMATOR OF THE MEAN ARE:">);
WRITE(LP[SPACE 2],<4(I2,X2,F13.9,X5)>, FOR R:=1 STEP 1 UNTIL N DO
[R,COEFF[R]]);
WRITE(LP,<"THE SUMS OF PAIRS OF COEFFICIENTS IN THE ESTIMATOR OF THE MEA
N ARE:">);
WRITE(LP[SPACE 2],<4(I2," AND ",I2,X2,F13.9,X5)>,FOR R:=1 STEP 2 UNTIL
((N-2) DIV 2) DO[R,R+1,COEFF[R]+COEFF[R+1]]);
WRITE(LP,<"THE SUMS OF QUADRUPLES OF COEFFICIENTS IN THE ESTIMATOR OF TH
E MEAN ARE:">);
WRITE(LP[SPACE 2],<4(I2," TO ",I2,X2,F13.9,X5)>,FOR R:=1 STEP 4 UNTIL
((N-6) DIV 2) DO[R,R+3,COEFF[R]+COEFF[R+1]+COEFF[R+2]+COEFF[R+3]]);
WRITE(LP[SPACE 2],<"VARIANCE OF ESTIMATOR OF MEAN IS VAR OF DISTRIB BY",
F13.9>,1/NORMALIZER);
SUMCOEFFS:=0;
FOR R:=1 STEP 1 UNTIL N DO SUMCOEFFS:=SUMCOEFFS+COEFF[R];
WRITE(LP,<"THE SUM OF THE COEFFICIENTS IN THE ESTIMATOR OF THE MEAN IS",
F13.9>,SUMCOEFFS);
WRITE(LP[SPACE 4],<"THE NUMBER OF DIGITS CORRECT BEFORE IMPROVEMENT IN
THE ESTIMATOR OF THE MEAN WAS",F9.5>,DIGITS);


COMMENT THE RANGE ESTIMATOR IS CALCULATED AND WRITTEN OUT;
SYMLINEQNIMPRV(N,COVARIANCE,MEAN,MEANVINV,DIGITSR,SINGMATRIX);
NORMALIZERR:=0;
FOR R:=1 STEP 1 UNTIL N DO NORMALIZERR:=*+MEANVINV[R]*MEAN[R];
FOR R:=1STEP 1 UNTIL N DO COEFFR[R]:=MEANVINV[R]/NORMALIZERR;
WRITE(LP,<"THE COEFFICIENTS OF THE BLCOS ESTIMATOR OF THE RANGE RATIO
ARE:">);
WRITE(LP[SPACE 2],<4(I2,X2,F13.9,X5)>,FOR R:=1 STEP 1 UNTIL N DO
[R,COEFFR[R]]);
WRITE(LP,<"VARIANCE OF ESTIMATOR OF RANGE IS VAR OF DISTRIB BY",F13.9>,
1/NORMALIZERR);
WRITE(LP[SPACE 3],<"THE NUMBER OF DIGITS CORRECT BEFORE IMPROVEMENT IN
THE ESTIMATOR OF THE RANGE WAS",F9.5>,DIGITSR);
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          COMMENT THE RESULTS ARE STORED ON DISK;
          FOR R:=1 STEP 1 UNTIL 8 DO PARTCOV1[R]:=COVARIANCE[1,R];
          FOR R:=2 STEP 1 UNTIL 7 DO PARTCOV2[R]:=COVARIANCE[2,R];
          FOR R:=3 STEP 1 UNTIL 6 DO PARTCOV3[R]:=COVARIANCE[3,R];
          FOR R:=4 STEP 1 UNTIL 5 DO PARTCOV4[R]:=COVARIANCE[4,R];
          READ(COUNTR8[0],1,COUNT);
          READ(CARD,<I2>,RERUN);
          IF RERUN=0 THEN COUNT[1]:=*+1;
          CONREGS[1]:=COUNT[1];
          CONREGS[2]:=N;
          CONREGS[3]:=REGS;
          WRITE(COUNTR8[0],1,COUNT);
          READ(RESULTS8[10*(COUNT[1]-1)],1,RUBBISH);
          WRITE(RESULTS8,3,CONREGS);
          WRITE(RESULTS8,(REGS+2) DIV 2,JUMP);
          WRITE(RESULTS8,(REGS+1) DIV 2,DENSA);
          WRITE(RESULTS8,(REGS+1) DIV 2,DENSB);
          WRITE(RESULTS8,1+(N+1) DIV 2,MEAN);
          WRITE(RESULTS8,(N+1) DIV 2,COEFF);
          WRITE(RESULTS8,SIZE(PARTCOV1),PARTCOV1);
          WRITE(RESULTS8,SIZE(PARTCOV2),PARTCOV2);
          WRITE(RESULTS8,SIZE(PARTCOV3),PARTCOV3);
          WRITE(RESULTS8,SIZE(PARTCOV4),PARTCOV4);
          WRITE(RESULTS8,1,RUBBISH);
          WRITE(LP[SKIP 1],<"THERE ARE NOW",I4," DISTRIBUTIONS STORED">,COUNT[1]);
          SINGMATRIX:END;
   END;
END.
```